NUMERICAL SIMULATION OF RECRYSTALLIZATION BEHAVIOUR OF Al-1%Mg DURING MULTI-PASS ROLLING

by

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DECLARATION

This is to certify that the results, calculations and other work presented in this thesis are essentially my own work, and that no part of it has been submitted for a degree at any other university.

Signed by candidate

D J Jansen

September 1994

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ABSTRACT

Microstructural changes which occur in aluminium alloys during rolling operations often play a large role in the performance of subsequent forming processes, such as deep-drawing and sheet-metal forming. It is becoming increasingly important to control these microstructural changes in order to optimise the subsequent processing route and thereby produce a cost-effective product. The ability to model the microstructural evolution of the alloy during rolling would greatly reduce the development cost and development time of a particular rolling schedule.

This thesis investigates the recrystallization behaviour of Al-1%Mg for a three pass rolling process. Equations describing the recrystallization process are implemented into a Finite Element code, and the effect of changing the roll speed is investigated. The results indicate that further work is required to empirically characterise the recrystallization behaviour of the particular alloy in question.

CONTENTS

CHAPTER 1

INTRODUCTION

1.1 Background

Rolling of aluminium alloys, whether hot, warm or cold, plays an important part in many manufacturing processes currently used. The rolled product is used either as is, or undergoes further processing such as deep-drawing or sheet metal forming. In each case, the rolling operation has a strong influence on the microstructure and properties of the final product, and the efficiency of any subsequent forming processes.

The microstructure of a material undergoes a number of changes during, and after, a high deformation cycle, termed the microstructural evolution of the material. The final microstructure is strongly dependent on parameters such as original microstructure, temperature, strain and strain rate. High deformation manufacturing processes can thus have a significant effect on microstructure due to the high strains, strain rates and temperature changes involved. A further phenomenon encountered during large deformation processes is that of Recovery and Recrystallization, in which the deformed microstructure is replaced with new, undeformed grains to relieve the internal plastic strain. Recovery refers to the microscopic rearrangements of dislocations within the material, while Recrystallization is the growth of new, unstrained grains, which generally nucleate at points of high strain and/or particles within the material. Recrystallization further implies that the new grains which are formed have a preferred orientation (or texture), which leads to anisotropic material behaviour. This anisotropic behaviour, once again, influences the effectiveness of the subsequent forming processes.

An understanding of the microstructural evolution which occurs during rolling is imperative in optimising the entire processing route, and has prompted a large research effort. The final microstructure of the rolled product is also dependent on various additional parameters, such as friction, interpass times, interpass temperatures and slab thicknesses. Thus the optimisation of a particular rolling process and schedule involves the interaction of process conditions, original microstructure and process induced changes to microstructure. The simulation of these interactions provides for many interesting research areas.

In aluminium alloys, the recrystallization characteristics and the material's texture are topics of current research. The simulation of microstructural evolution during rolling has also received a large amount of attention, since a successful model would make the investigation of various parameters more cost effective than actual rolling trials. However, since many parameters are involved, the development and validation of such a model requires considerable effort. Due to the advances in computer technology, it is now possible to incorporate the well-defined parameters into a single model, and to study the microstructural evolution for various conditions.

The aim of this work is to develop a model which simulates the recrystallization behaviour of an aluminium alloy during multipass rolling. This is achieved by implementing the Sellars equations[1,2] into a general purpose Finite Element program ABAQUS[27] via a user subroutine.

1.2. Recovery and Recrystallization

The kinetics of recrystallization for C-Mn steel have been studied by Sellars and Whiteman [1,2], and implemented by Benyon and Sellars [3] into a computer simulation package (SLIMMER). Temperature effects were studied by Sheppard and Wright [4] and by Yiu *et al* [5], while the development of the microstructure within the roll-gap was reported by Zaidi and Sheppard [6]. Other studies considered the determination of material constants for recrystallization equations for various aluminium alloys[7-15]. The details of the equations are discussed in chapter 2.

The recrystallisation behaviour of a material is governed strongly by the temperature of the rolling process, and a temperature compensated strain rate (the Zener-Hollomon parameter (Z)). Other important parameters are the original grain size and the plastic strain induced by the deformation. Recrystallization leads to a decrease in retained strain in the material, and thus softening and a decrease in the flow stress, which effects any subsequent forming processes.

Two types of Recovery and Recrystallization have been identified. During the deformation, these are termed dynamic recovery and recrystallization, and after the deformation, these are termed static recovery and recrystallization. For Aluminium, however, dynamic recovery occurs as soon as dislocations appear, since self-diffusion is promoted at high temperatures, and is sufficient to prevent any further work hardening. This phenomenon prevents dynamic

recrystallization from occurring to any large extent. The static effects are far greater in this particular case, and it is for this reason that the work presented here has not taken the dynamic effects into account.

1.3. Texture

The Texture of a material refers to the preferred orientation of the grains within the material. If a material has a strong texture, then it will be strongly anisotropic. This can have detrimental effects during processes such as deep-drawing and sheet metal forming. A major research effort has gone into the analysis of textures, which are usually measured with the aid of X-ray devices, and computational methods of obtaining these textures.

Texture development occurs in two distinct processes during rolling, namely deformation texture and recrystallization texture. Numerous studies have been done on deformation textures [16-18], and numerical simulation routines have been successfully implemented[19-21]. This texture develops through twinning and slip within the existing material, and effectively brings about a change of the preferred orientation of the grains within the material. This does not occur by the grain physically rotating, but rather by the structures within a particular grain changing to yield a new preferred orientation.

Deformation texture is relatively well understood, unlike recrystallization texture. The evolution of texture during recrystallization is controversial and can be divided into two schools of thought, viz. oriented growth and oriented nucleation [22]. Oriented growth assumes a nucleus, and the subsequent growth rates determine which of the various orientations survive, while oriented nucleation suggests that the orientation is fixed from the beginning.

Depending on the material used and the test conditions, evidence for both can be found, but as yet no conclusive mechanisms satisfying all the evidence has been suggested. This has prompted the formation of an international panel to investigate this controversy [22]. While attempts have been made to model the evolution of recrystallization texture [23-25], the results cannot be taken to be true for every case. Without extensive experimental metallographic research into a particular material, it is impossible to predict the texture resulting from a rolling process involving recrystallization.

Numerical modelling of texture has, thus far, concentrated on modelling the orientation distribution diagrams (ODF) associated with a particular texture. These diagrams are cumbersome and difficult to understand (Figure 1.1), and the translation of this information into something more manageable, for example volume fractions, has not been achieved with numerical modelling. This will only be possible once the controversy described above is resolved.

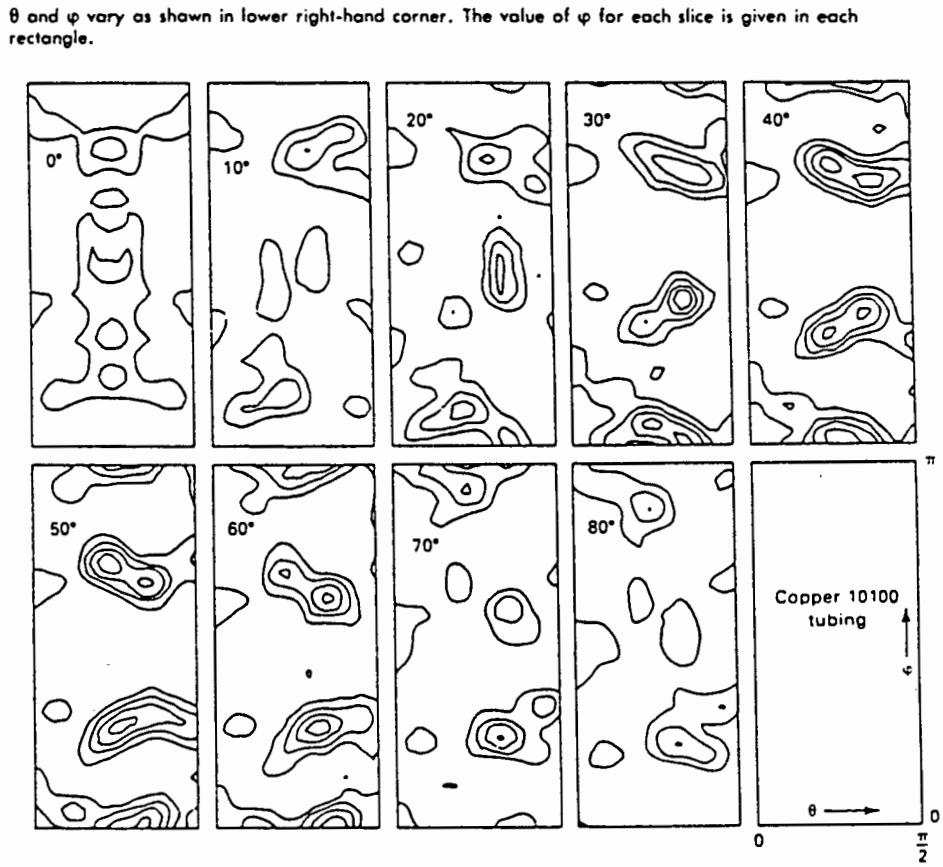


Figure 1.1. Orientation Distribution Diagram for Copper 10100 tubing.

1.4. Modelling of Recrystallization

Using the Sellars equations, several researchers [13,14] have modelled the recrystallization of aluminium using Finite Element, Finite Difference, or simple single point methods. In these models, the effects of temperature, strain, and strain rate have been studied, with an emphasis on the Zener-Hollomon parameter. It has been found that differences in the Zener-Hollomon parameter exist in the through-thickness of a rolled strip, which make single point type analyses inadequate. Finite Element Modelling is useful in this regard since it is able to perform the recrystallization calculations at various points through out the mesh. For the purposes of this work, the Recrystallization routines have been incorporated into a standard von Mises elastic-plastic material model, described in Chapter 2. The Finite Element Modelling of the recrystallization process is discussed in detail in Chapter 4.

1.5. Validation

While the Sellar's recrystallization equations have been determined empirically, the model's method of measuring strain rate has not been verified through experiment. Likewise, the grain size averaging scheme used here is theoretical, and does not cater adequately for multipass rolling. This model thus supplies results which are only qualitative, and the accuracy of the results can only be determined by comparison with empirical data, which is not available at present. However, once this data is available, the full model can be adjusted to accurately predict mean grain sizes.

Furthermore, the material used for this research, namely Al-1%Mg, has been chosen since data for this material are readily available. In order to apply this model to any other material, it is necessary to fully characterise the material by doing extensive empirical testing.

CHAPTER 2

THE MATERIAL MODEL

2.1. Elastoplasticity

The basic assumption of Elastoplasticity is that the material behaves in an entirely elastic manner until some yield condition is met. The elastic stress-strain relationship is described by

$$\underline{\sigma} = \underline{\underline{D}}^{el}\,\underline{\varepsilon} \tag{2.1.1}$$

where $\underline{\sigma}$ and $\underline{\varepsilon}$ are the stress and strain tensors respectively and $\underline{\underline{D}}^{el}$ is the elastic tensor for an isotropic material. The strain tensor can be decomposed into elastic and plastic components once yielding has occurred, and can be written as

$$\underline{\varepsilon} = \underline{\varepsilon}^{el} + \underline{\varepsilon}^{pl} \tag{2.1.2}$$

where $\underline{\varepsilon}^{el}$ and $\underline{\varepsilon}^{pl}$ are the elastic and plastic components of strain respectively.

The stress and strain tensors can be represented by deviatoric quantities given by

$$\begin{aligned} \underline{S} &= \underline{\sigma} - \frac{1}{3}(trace\,\underline{\sigma})\underline{\underline{I}} \\ \underline{e} &= \underline{\varepsilon} - \frac{1}{3}(trace\,\underline{\varepsilon})\underline{\underline{I}} \end{aligned} \tag{2.1.3}$$

where $\underline{S}$ is the deviatoric stress, $\underline{e}$ is the deviatoric strain and $\underline{\underline{I}}$ is the identity tensor. The deviatoric strain can also be decomposed into elastic and plastic components, and the stress-strain relationship for elastic behaviour is given by

$$\underline{S} = 2G\underline{e}^{el} \tag{2.1.4}$$

where G is the shear modulus and $\underline{e}^{el}$ is the elastic deviatoric strain. The volumetric strain, ε_V^{el}, is given by

$$\varepsilon_V^{el} = trace\,\underline{\varepsilon} \tag{2.1.5}$$

The model assumes that all volumetric behaviour is purely elastic, i.e. $\varepsilon_V^{pl} = 0$, and thus plastic deformation is not associated with a change in volume.

2.2. Yield Condition

The material model utilises the standard von Mises yield criterion for isotropic hardening materials. Initial yielding occurs when the applied von Mises stress is greater than the static yield stress, i.e.

$$\dot{\varepsilon}^{pl} \neq 0 \quad \text{for} \quad q \geq \sigma_y \tag{2.2.1}$$

$$q = \sqrt{\frac{3}{2}\underline{S}:\underline{S}} \tag{2.2.2}$$

where $\dot{\varepsilon}^{pl}$ is the plastic strain rate, σ_y is the static yield stress and q is the von Mises stress (predicted).

Once yielding has occurred, the applied stress q must satisfy the yield condition

$$q = \overline{\sigma} \tag{2.2.3}$$

given by

$$\overline{\sigma} = 2G\tilde{e} - 3G\Delta\bar{e}^{pl} \tag{2.2.4}$$

where G is the shear modulus, $\tilde{e}$ is the equivalent total strain, and $\Delta\bar{e}^{pl}$ is the equivalent plastic strain.

2.3. Strain Rate Dependence

Strain rate dependence has the effect of increasing the yield strength of a material, during deformation, with increasing strain rate. The rate dependence model used is described by

$$\dot{\varepsilon}^{pl} = D\left(\frac{\bar{\sigma}}{\sigma_y} - 1\right)^p \tag{2.3.1}$$

where $\dot{\varepsilon}^{pl}$ is the equivalent plastic strain rate, $\bar{\sigma}$ the applied stress, and D and p are temperature dependant material constants. The values used for D and p were approximated as described in Appendix I.

2.4. Recrystallization Behaviour

The recrystallization behaviour of the material in question, namely Al-1%Mg, was studied by Sellars, Irisarri and Puchi [7] using total equivalent strain and total equivalent strain rate. For large deformation metal plasticity, the equivalent elastic strain and equivalent elastic strain rate can be neglected, and the Zener-Hollomon parameter Z at a material point can be defined by

$$Z = \dot{\bar{\varepsilon}}^{pl} e^{\frac{Q_{def}}{RT}} \tag{2.4.1}$$

where R is the universal gas constant, Q_{def} is the activation energy for deformation, T is the absolute temperature, and $\dot{\bar{\varepsilon}}^{pl}$ is the time average equivalent plastic strain rate, calculated by

$$\dot{\bar{\varepsilon}}^{pl} = \frac{\sum \Delta \bar{e}^{pl}}{\sum \Delta t} \tag{2.4.2}$$

and $\sum \Delta \bar{e}^{pl}$ is the equivalent plastic strain summed over the entire deformation cycle, and $\sum \Delta t$ is the total time of deformation.

The time to 50% recrystallized $(t_{0.5})$ is given by

$$t_{0.5} = \alpha d_0^{1.35} \varepsilon^{-2.7} Z^{-1.1} e^{\frac{Q_{rex}}{RT}} \tag{2.4.3}$$

where d_0 is the original grain size in μm, ε^{pl} is the equivalent plastic strain, Q_{rex} the activation energy for recrystallization, and α a material constant.

The recrystallized grain size (d_{rex}) is defined as

$$d_{rex} = \beta d_0^{1.3} \left(\varepsilon^{pl}\right)^{-0.39} Z^{-0.24} \tag{2.4.4}$$

where β is a material constant. The recrystallized volume fraction (X_V), associated with a material point, is given by

$$X_V = 1 - e^{\left(-0.693\left(\frac{t}{t_{0.5}}\right)^n\right)} \tag{2.4.5}$$

where n is a material constant. The values of these constants are listed in Appendix I.

The grain size evolution during recrystallization of a single volume fraction is described in terms of a recrystallizing volume fraction and a sacrificial volume fraction, with grain sizes d_r and d_S respectively. The recrystallizing grains grow at the expense of the sacrificial grains, as given by

$$d_r = X_v^{\frac{1}{3}} d_{rex} \tag{2.4.6}$$

and

$$d_s = (1 - X_v) d_0 \tag{2.4.7}$$

The mean grain size (d_{mean}) is an average of these quantities, given by

$$d_{mean} = X_v d_r + (1 - X_v) d_s \tag{2.4.8}$$

By substituting Equations 2.4.6 and 2.4.7 into Equation 2.4.8, d_{mean} is expressed in terms of d_0 and d_{rex} as shown below for a single deformation cycle (roll pass)

$$d_{mean} = X_V^{\frac{4}{3}} d_{rex} + X_0^2 d_0 \tag{2.4.9}$$

where X_0 is the unrecrystallized volume fraction given by

$$X_0 = 1 - X_v \tag{2.4.10}$$

The grain size evolution of a material point, with a temperature of 623K and an original grain size of $100\mu m$ is shown in Figure 2.1 below. The volume fraction recrystallized is also shown in this figure.

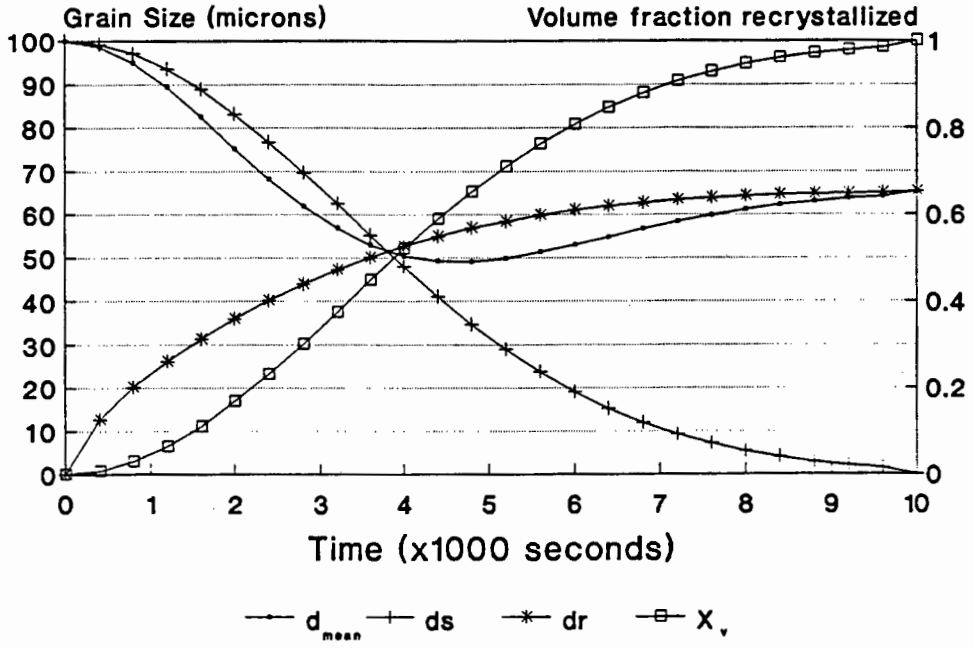


Figure 2.1. Grain Size evolution and volume fraction recrystallized as a function of time.

2.5. Grain Size Evolution in Multipass Rolling

The expressions introduced above are true for the first roll pass, but for the second and subsequent roll passes the calculation of the mean grain size is more complex. The values of d_0 for the second roll pass are given by

$$d_0(1) = d_s$$
$$d_0(2) = d_r \qquad (2.5.1)$$

where d_s and d_r are the sizes of the sacrificial and recrystallizing grains as calculated at the end of the first recrystallization phase. The values of $t_{0.5}(i)$ and $d_{rex}(i)$, where $i = 1,n$, are calculated as shown by Equations 2.4.3 and 2.4.4 respectively, using the $d_0(i)$ values from Equation 2.5.1 and plastic strain values calculated during the second deformation. The plastic strain values are cumulative, thus the previously unrecrystallized material will have a higher strain value than the previously recrystallized material.

Implementation of Equation 2.4.9 assumes that the material recrystallizes at the expense of the unrecrystallized volume fraction. This equation cannot be applied to multipass rolling in its present form, since it would assume that the recrystallization process in each sacrificial volume fraction occurs independently of any other process, i.e. $X_0(1)$ serves as a sacrificial volume fraction for $X_V(1)$ only, $X_0(2)$ serves as a sacrificial volume fraction for $X_V(2)$ only, and so on, which is not physically meaningful. This approach also results in numerical continuity problems when tracking the average grain size over more than one deformation cycle.

The proposed approach alleviates these problems by treating the original material ($X_0(1)$) as the only sacrificial volume fraction, and all other volume fractions as ones which arise from it. The values of $X_V(i)$ and $X_0(i)$ are calculated as shown in Equations 2.4.5 and 2.4.10 respectively, and values of $d_r(i)$ and $d_s(i)$ are calculated, to serve as initial grain size values ($d_0(i)$) values for the subsequent deformation cycle, as shown in Equations 2.4.6 and 2.4.7 respectively. The proposed approach modifies Equation 2.4.9 as follows

$$d_{mean} = X_V(i)^{4/3} d_{rex}(i) + X_0(j)^{4/3} d_0(j) + X_0(1)^2 d_0(1) \qquad (2.5.2)$$

where
$$i = 1,n$$
$$j = 2,n$$

and n indicates the number of roll passes.

It should be noted here that Equations 2.4.9 and 2.5.2 are entirely theoretical, and are used here in the absence of empirically-based formulae. Future research should include a testing program to establish a grain size averaging scheme which is physically meaningful.

2.6. Stress and Strain update due to Recrystallization

In multipass rolling, the above relationships lead to a number of different volume fractions with differing amounts of residual strain, i.e. internal plastic strain, since the recrystallized portion of the material is assumed to be strain free. This results in a decrease in the amount of total residual strain present in the material, which is termed strain relaxation for the purposes of this research. The formation of various volume fractions during multipass rolling is shown in Figure 2.2 below.

Note that the term "residual strain" refers to the amount of internal plastic strain which is retained in the material after recrystallization, and is calculated by

$$\varepsilon_{res}^{pl} = \sum_{1}^{k} X_0(i).\varepsilon^{pl}(i) \tag{2.6.1}$$

where $X_O(i)$ are the unrecrystallized volume fractions, $\varepsilon^{pl}(i)$ are the corresponding calculated strain values, and k indicates the number of volume fractions. The resulting yield strength is given by

$$q = \sigma_y + H\varepsilon_{res}^{pl} \tag{2.6.2}$$

where H is the tangent to the stress-plastic strain curve.

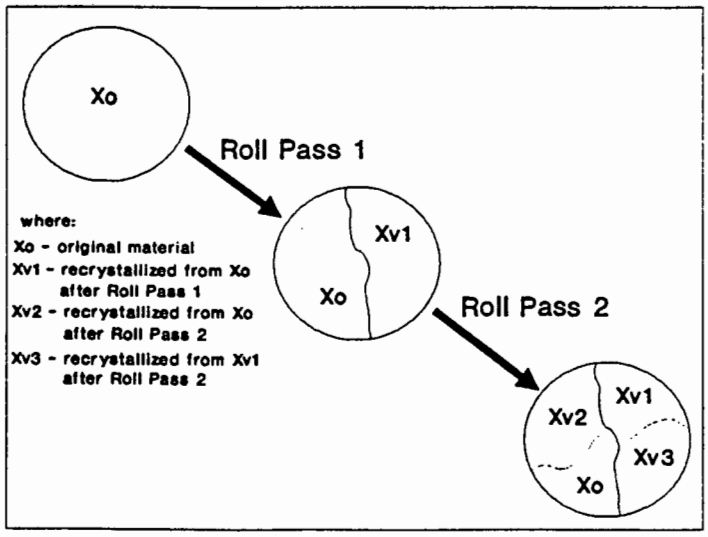


Figure 2.2. Diagram describing the evolution of different volume fractions during multipass rolling.

CHAPTER 3

RECRYSTALLIZATION ROUTINES: TEMPERATURE AND INTERPASS TIME EFFECTS.

3.1. Description

The formulation of the recrystallization behaviour was tested separately for verification purposes, in which the effects of temperature and interpass times on average grain size were investigated. The roll schedules used for these tests are described in the relevant sections. Material constants for Al-1%Mg, as described in Appendix I, were used.

The recrystallization equations (2.4.1 - 2.5.2) are solved for assuming that the equivalent plastic strain rate for each roll pass is given to simulate the recrystallization behaviour at a representative volume around an arbitrary material point.

3.2. Temperature

The roll schedule used for this comparison is shown in Table 3.1 below. The temperature was held constant over the three roll passes, and analyses were performed at 523, 623 and 673K. The original grain size for all analyses was $100\mu m$ and the interpass times, i.e. the time between roll passes, was 1000s. An equivalent plastic strain rate of $13.75s^{-1}$, corresponding to a roll speed of 240m/min, was used for the analyses.

ROLL PASS	STARTING GAUGE (mm)	FINAL GAUGE (mm)	RECRYSTALLIZATION TIME (S)
1	60	50	1000
2	50	40	1000
3	40	30	1000

Table 3.1. Rolling Schedule used for temperature study.

Figure 3.1 shows the average grain size (d_{mean}), as a function of time, at the three different temperatures. It can be seen that the average grain size is affected by temperature changes, with a higher temperature causing faster recrystallization and larger grain sizes. This is a consequence of the dependence of both the Zener-Hollomon parameter and the time to 50% recrystallization on temperature.

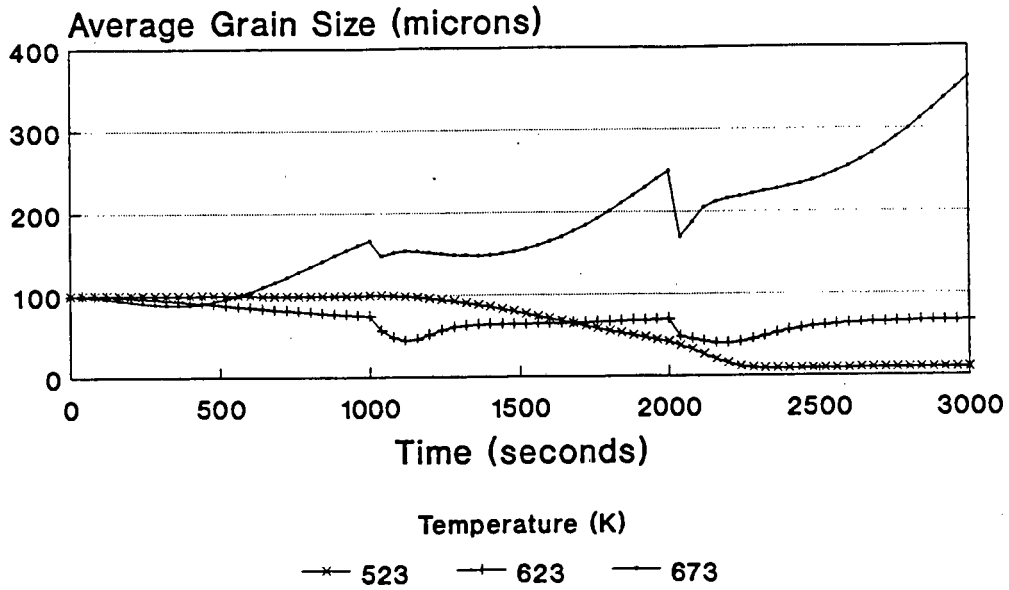


Figure 3.1. Plot of average grain size (d_{mean}) versus time at three different temperatures.

3.3. Interpass Times

The roll schedule for this study is shown in Table 3.2 below. All values are the same as those for the temperature study, except that the temperature was held constant at 623K, and the interpass times were varied as shown below. Three analyses were performed with interpass times of 100, 500 and 1000 seconds. The total time was kept at 3000s for the purposes of comparison.

ROLL PASS	STARTING GAUGE (mm)	FINAL GAUGE (mm)	TEMPERATURE (K)
1	60	50	623
2	50	40	623
3	40	30	623

Table 3.2. Rolling schedule used for interpass time study.

Figure 3.2 below shows the effect of varying interpass times on the average grain size (d_{mean}). The slow interpass times lead to little recrystallization after the first pass, but recrystallization is rapid after the second pass. This is due to the residual strain in the material, which is a driving force for recrystallization and is cumulative.

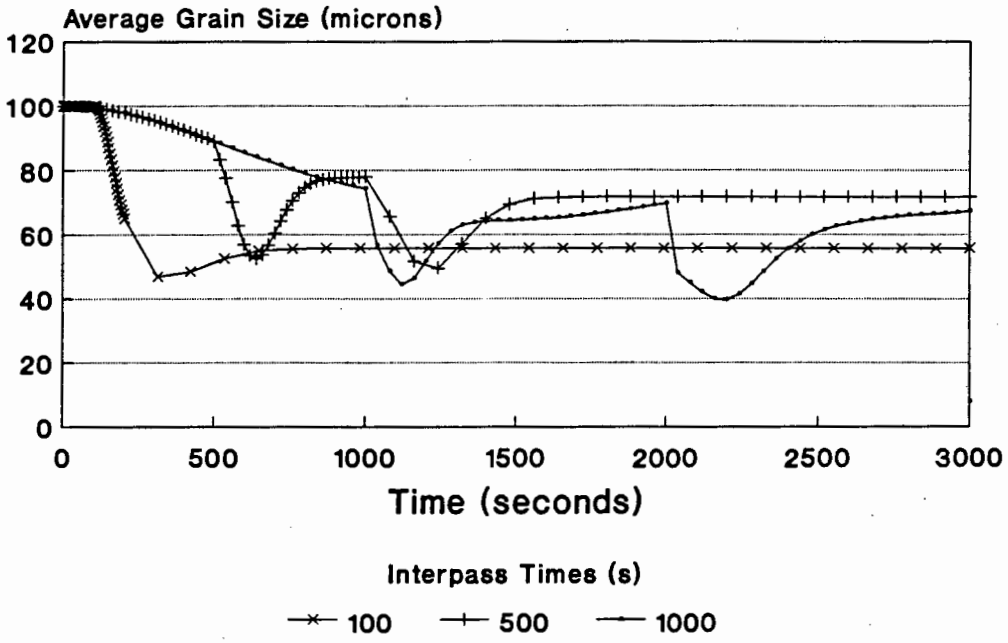


Figure 3.2.Effect of varying interpass times on average grain size (d_{mean}) over three roll passes.

CHAPTER 4

FINITE ELEMENT MODEL

The implementation of the plasticity model together with the recrystallization model into ABAQUS is described in this chapter. The code for this user material is included in Appendix II. The procedure adopted for calculating the elastic-plastic response of the material is the standard backward Euler integration scheme with a radial return stress update procedure [26].

4.1. Basic Steps

As the recrystallization equations are closely coupled to the elastic-plastic material model, the purpose of the user material, apart from modelling recrystallization, is to define the behaviour of the material under an applied loading condition. The user subroutine is required to calculate the stress state, consistent modulus, and state variables at the end of a given increment, given the stress state, strain and state variables at the beginning of the increment, and the increment in strain. This is done at each Gauss point. If the material deforms plastically, the user subroutine is required to calculate the elastic and plastic components of strain as defined in Chapter 2. The basic solution procedure consists of the following steps:

1. *Elastic Predictor.* The elastic predictor is calculated by assuming the change in strain is purely elastic.

2. *Yield Condition.* The yield condition, described in Section 2.2, is applied to the predicted stress state. If yielding has not taken place then the predicted stress state is taken as the true stress state. However, if yielding has occurred, then the plasticity algorithm is employed to calculate the true stress state.

3. *Von Mises Plasticity Algorithm.* Plasticity calculations are performed to calculate the increment in plastic strain, the stress state at this increment, and the value of the state variables. It is during these calculations that the strain rate used for the recrystallization procedure is calculated, as described in Section 2.4.

4. *Consistent Modulus.* The terms within the Consistent Modulus are updated, according to the values calculated by the plasticity algorithm, and implemented in the iterative solution of the global finite element equations.

4.2. Elastic Predictor

Given the increment of strain and the total strain at the beginning of an increment, the elastic predictor $\underline{\sigma}^{el}$ is calculated by

$$\underline{\sigma}^{el} = \underline{\underline{D}}^{el}\left(\underline{\varepsilon}^{el}\Big|_t + \Delta\underline{\varepsilon}\right) \tag{4.1.1}$$

where $\underline{\underline{D}}^{el}$ is the elastic constitutive matrix, $\underline{\varepsilon}^{el}\Big|_t$ is the elastic strain at the beginning of the increment, and $\Delta\underline{\varepsilon}$ is the increment in strain. The deviatoric elastic stress predictor $\underline{S}^{el}$ is given by

$$\underline{S}^{el} = \underline{\sigma}^{el} - \frac{1}{3}trace\left(\underline{\sigma}^{el}\right)\underline{\underline{I}} \tag{4.1.2}$$

where $\underline{\underline{I}}$ is the identity tensor. The yield condition described in Chapter 2 is then applied to the deviatoric elastic stress predictor, i.e.

$$\dot{\varepsilon}^{pl} \neq 0 \quad \text{for} \quad q \geq \overline{\sigma} \tag{4.1.3}$$

If the yield condition is true, then the increment in plastic strain is calculated as described below.

4.3. Von Mises Plasticity Algorithm

The deviatoric stress can be written as

$$\underline{S} = 2G\left(\underline{e}^{el}\Big|_t + \Delta\underline{e} - \Delta\overline{e}^{pl}\,\underline{n}\right) \tag{4.1.4}$$

where G is the elastic shear modulus, $\Delta\bar{e}^{pl}$ the plastic multiplier and $\underline{n}$ the flow direction, defined as

$$\underline{n} = \frac{3}{2}\frac{\underline{S}}{q} \tag{4.1.5}$$

where q is defined as before (Chapter 2). For rate dependent materials, $\bar{\sigma}$ is defined as

$$\bar{\sigma} = \sigma_0\left(1+\left(\frac{\Delta\bar{e}^{pl}}{D\Delta t}\right)^{\frac{1}{p}}\right) \tag{4.1.6}$$

where σ_0 is the current yield stress given by

$$\sigma_0 = \sigma_y + H\left(\bar{e}^{pl}\Big|_t + \Delta\bar{e}^{pl}\right) \tag{4.1.7}$$

where σ_y is the static yield stress, $H = \frac{\partial\sigma}{\partial\bar{e}^{pl}}$, and $\bar{e}^{pl}\Big|_t$ is the equivalent plastic strain at the beginning of the increment. The resulting equation for the updated stress state is

$$\underline{S} = \frac{2G}{\left(1+\frac{3G}{q}\Delta\bar{e}^{pl}\right)}\hat{\underline{e}} \tag{4.1.8}$$

where

$$\hat{\underline{e}} = \underline{e}^{el}\Big|_t + \Delta\underline{e} \tag{4.1.9}$$

and $\hat{\underline{e}}$ is known from the total strain increment, and $\Delta\bar{e}^{pl}$ and q must be solved for.

Taking the inner product of equation 4.1.8 with itself gives

$$q + 3G\Delta\bar{e}^{pl} = 2G\tilde{e} \tag{4.1.10}$$

where $\tilde{e}$ is the equivalent strain, defined as

$$\tilde{e} = \sqrt{\frac{3}{2}\hat{\underline{e}}:\hat{\underline{e}}} \tag{4.1.11}$$

Imposing the yield condition on the uniaxial form of the yield behaviour gives

$$G\left(2\bar{e}-3\Delta\bar{e}^{pl}\right)-\bar{\sigma}=0 \tag{4.1.12}$$

which is a non-linear equation for $\Delta\bar{e}^{pl}$ in rate dependent problems where $\bar{\sigma}$ is dependent on the equivalent plastic strain. A Newton iterative scheme is used to solve for $\Delta\bar{e}^{pl}$ in 4.1.12

$$\Delta\bar{e}_n^{pl}=\Delta\bar{e}_{n-1}^{pl}+\frac{G\left(2\bar{e}-3\Delta\bar{e}_{n-1}^{pl}\right)-\bar{\sigma}}{3G+H} \tag{4.1.13}$$

where n denotes the iteration number, and the result is then substituted into equation 4.1.8. to give the updated stress state. Note that when the yield condition is satisfied, $q\equiv\bar{\sigma}$ in 4.1.8, and strain can be obtained directly.

4.4. Basic Finite Element Equations

The formulation used in this analyses involves the solution of the force equilibrium equation

$$\int_V \underline{\sigma}dV+\int_S \underline{f}dS=0 \tag{4.1.14}$$

where $\underline{f}$ is the surface traction tensor and $\underline{\sigma}$ is the stress tensor at a material point. The basic finite element equation is derived from the Principle of Virtual Work to give

$$F(\underline{u},\delta\underline{u})=\int_V \delta\underline{\varepsilon}^T\,\underline{\sigma}dV-\int_S \delta\underline{u}^T\,\underline{f}dS \tag{4.1.15}$$

where δ are virtual quantities and $\underline{u}$ is the displacement vector. By applying a Taylor series expansion to equation 4.1.15, an iterative procedure to solve for increments in displacement can be obtained

$$F_n^i(\underline{u}+d\underline{u},\delta\underline{u})=F_n^i(\underline{u},\delta\underline{u})+\left.\frac{\partial F}{\partial\underline{u}}\right|_n^i\partial\underline{u}=0 \tag{4.1.16}$$

where n denotes increment n and i denotes iteration i. The iterative procedure in equation 4.1.16 can be written in the standard form for finite element calculations

$$\underline{\underline{K}}\Big|_n^i d\underline{u}_n^i = \underline{F}_n^i \tag{4.1.17}$$

where $d_n^i\underline{u}$ is the nodal displacement vector, and $\underline{\underline{K}}\Big|_n^i$ is the stiffness matrix, defined as

$$\underline{\underline{K}}\Big|_n^i = \frac{\partial F}{\partial \underline{u}} = \int_V \underline{\underline{B}}^T \underline{\underline{D}}\Big|_n^i \underline{\underline{B}} dV \tag{4.1.18}$$

where $\underline{\underline{B}}$ is the standard strain-displacement matrix, and $\underline{\underline{D}}$ is the consistent tangent modulus given by

$$\underline{\underline{D}}\Big|_n^i = \frac{\partial \underline{\sigma}}{\partial \underline{\varepsilon}}\Big|_n \tag{4.1.19}$$

and $\underline{F}\Big|_n^i$ is the appropriate residual load vector.

4.5. Consistent Tangent Modulus.

Once the stress state has been updated, the terms for the consistent tangent modulus can be updated. The general equation is derived elsewhere [27], and is defined in terms of total stress increments versus total strain increments, i.e.

$$\partial\underline{\sigma} = \left[Q\underline{\underline{\mathfrak{I}}} + \left(K - \frac{1}{3}Q \right) \underline{\underline{I}}\,\underline{\underline{I}} - R\underline{S}\,\underline{S} \right] : \partial\underline{\varepsilon} \tag{4.1.20}$$

where K is the bulk modulus, $\underline{\underline{\mathfrak{I}}}$ is the fourth order unit tensor and $\underline{\underline{I}}$ is the identity tensor.

The constants Q and R are given by

$$Q = \frac{q}{\bar{e}} \tag{4.1.21}$$

and

$$R = \frac{3\left(1 - \Delta\bar{e}^{pl}\dfrac{H}{q}\right)}{2q\bar{e}(1+B)} \tag{4.1.22}$$

where $$B = \frac{H}{3G} \tag{4.1.23}$$

CHAPTER 5

ROLL PASS SIMULATION - TEST CASE

5.1. Purpose

A test case was implemented to confirm that the recrystallization routines have been incorporated into the finite element code correctly. This was done by allowing long recrystallization times after each roll pass. The rolling schedule is shown in Table 5.1 below, and is similar to that used in Chapter 3.

ROLL PASS	STARTING GAUGE (mm)	FINAL GAUGE (mm)	STANDING TIME (s)
1	60	50	1000
2	50	40	1000
3	40	30	1000

Table 5.1. Rolling Schedule of Test Case

The test case was analysed using a constant roll speed of 240m/min (circumferential speed), and a constant temperature of 623K. The initial grain size was assumed to be $100 \mu m$. The recrystallization times were taken as 1000 seconds after each roll pass, compared to a more realistic time of 100 to 500s.

5.2. Geometry and Mesh

A two-dimensional analysis was used to model the rolling process, assuming plane strain conditions. Plane strain linear elements with reduced integration were used. The mesh was graduated towards the surface, thus refining the elements at the surface of the slab. This was done to improve the accuracy of the results, since the surface elements would be deformed the most. Contact elements were placed on all the free surfaces of the slab to simulate contact and friction conditions between the roll and the slab.

The roll itself was modelled as a rigid body. Taking advantage of symmetry about the y-axis, it was only necessary to model the upper roll and half of the slab. The actual rolling operation was performed by specifying a rotation of the roll and a suitable friction coefficient to pull the slab through the roll gap.

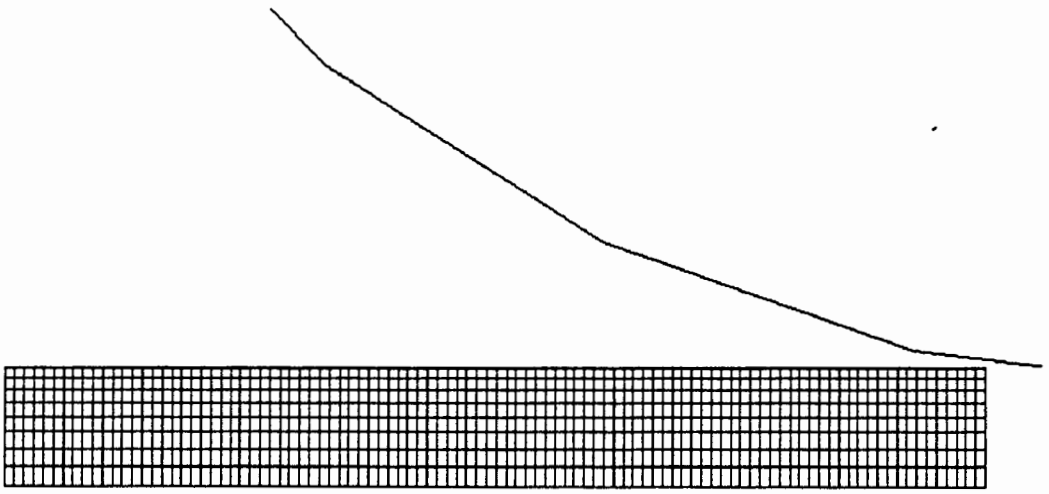


Figure 5.1. Schematic diagram of the mesh and the lower portion of the roll. The slab is 0.25m in length and the roll radius is 0.375m.

5.3. Number of Roll Passes

At present, the model is limited to three, or less, roll passes. It may be desirous to model more roll passes, depending on the particular analysis. To modify the present UMAT to accommodate further roll passes would require increasing the amount of solution dependent variables. This is a relatively simple modification, and would not require a great effort. However, each roll pass which is added must be described by at least five steps in the ABAQUS input deck (see Appendix

III), and most of the boundary conditions are defined by doing some runs to determine the required displacements, angular displacement of the roll, etc. The three roll pass analysis consisted of 14 steps, with an approximate CPU time requirement of 515 minutes on an IBM RISC 6000 560 workstation. Due to the CPU requirement of these runs, the process of defining the additional roll passes is a time consuming exercise. A description of the various Solution Dependent Variables can be found in Appendix IV.

5.4. Contact and Friction

Contact was defined by adding specialised contact elements on the slab. These contact elements are governed by a contact pressure between the surfaces. For rigid surfaces, the contact follows a 'hard' contact condition, described by

$$p = 0 \quad \text{for} \quad h < 0$$

and

$$p = \bar{k}h, \text{ with } \bar{k} \text{ infinite, for } h \geq 0 \tag{5.4.1}$$

where p is the contact pressure and h is the 'over closure' between the contact elements and the rigid surface.

Friction is modelled using a standard Coulomb friction model which is provided in ABAQUS. This model assumes that no relative motion occurs if the equivalent frictional stress, τ_{eq}, is less than the critical stress, τ_{crit}, where

$$\tau_{eq} = \sqrt{\tau_1^2 + \tau_2^2} \tag{5.4.2}$$

and

$$\tau_{crit} = \mu p \tag{5.4.3}$$

where τ_1 and τ_2 are the planar frictional stress components, μ is the friction coefficient, and p is the contact pressure.

5.5. End Effects

End effects, resulting in large mesh distortions, occur when the reduction in the gauge of the slab becomes large enough that the quadrilateral elements at the extreme ends of the slab become extremely distorted and collapse into triangles, i.e. two sides lie flat along the top edge of the slab. When these elements subsequently come into contact with the roll (rigid surface), numerical problems are encountered which can extend some distance towards the mid section of the slab. While the ends of the slab are not important from a results point of view, it is important to minimise these effects to ensure that the results taken from the centre portion of the slab are accurate. Figure 5.2 shows a mesh plot of some severe end-effects. To avoid these excessive distortions, rezoning the mesh at regular intervals would have to be implemented.

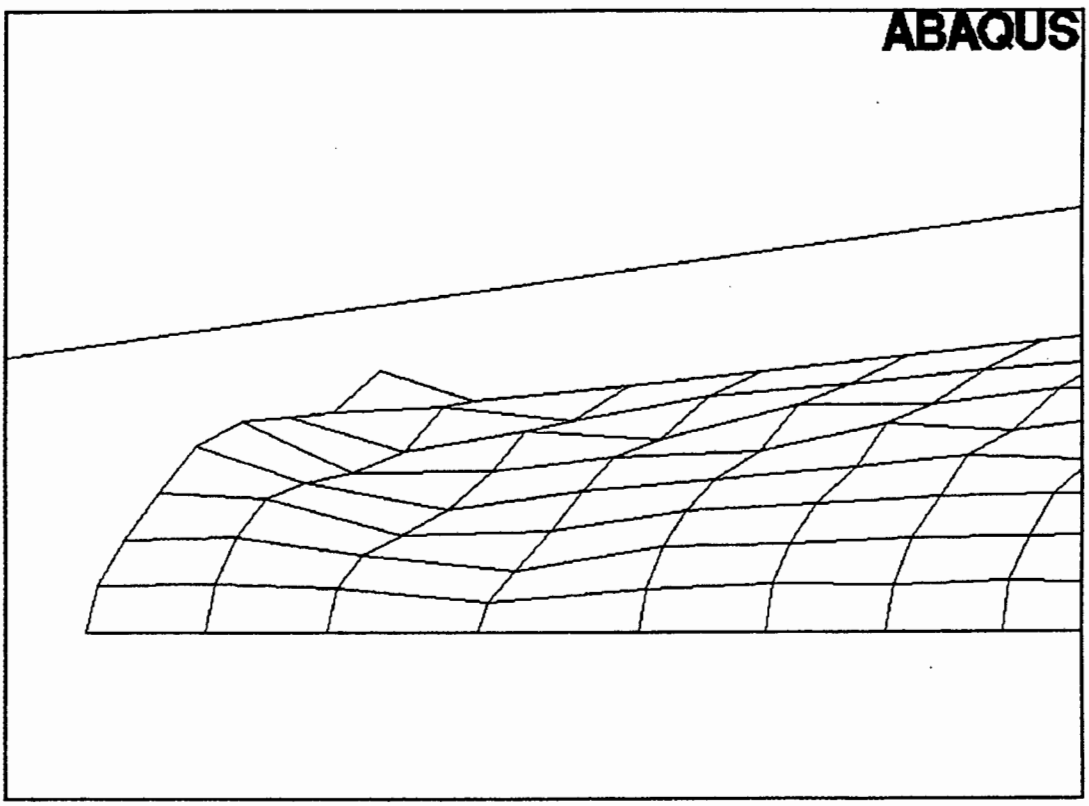


Figure 5.2. Mesh plot showing severe distortions.

5.6. Discussion of Results

History plots of residual plastic strain, average grain size and volume fraction recrystallized were generated at mid-section of the slab. The change in the Zener-Hollomon parameter through the thickness of the slab after each roll pass was also investigated. Note that the first roll pass occurs at $t = 0$, resulting in non-zero values of strain at this point. Each of the above-mentioned variables will be discussed below.

5.6.1. Zener-Hollomon Parameter

Figure 5.3 shows the Zener-Hollomon parameter through the thickness of the slab after each roll pass. For purposes of comparison, the positions through the thickness have been normalised, with 1 representing the centre, and 0 representing the surface. The Zener-Hollomon parameter is proportional to the strain rate (Equation 2.3.1), and the figure consequently shows that the strain rate is higher at sub-surface positions than it is at the surface and, as the slab gets thinner, the strain rate becomes larger, even though the roll speed is constant. Figure 5.4 relates this behaviour to the contours of the Zener-Hollomon parameter at the central portion of the slab after the third roll pass. This shows a banded structure, which is expected in the 'steady-state' region of the rolling model.

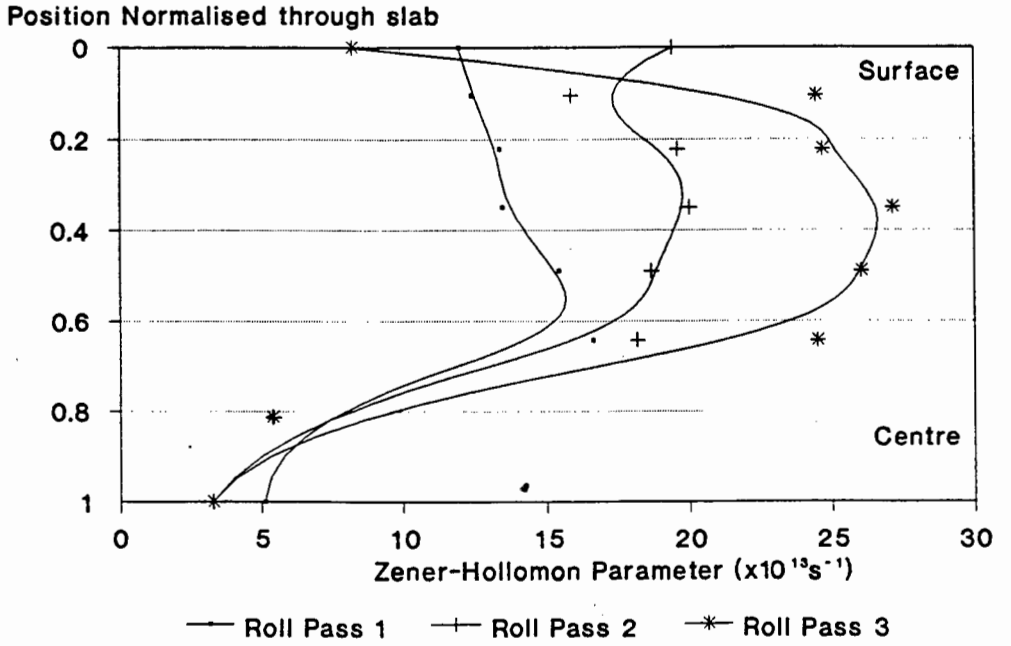


Figure 5.3. Zener-Hollomon parameter through the thickness of the material after each roll pass. Positions have been normalised for the purposes of comparison.

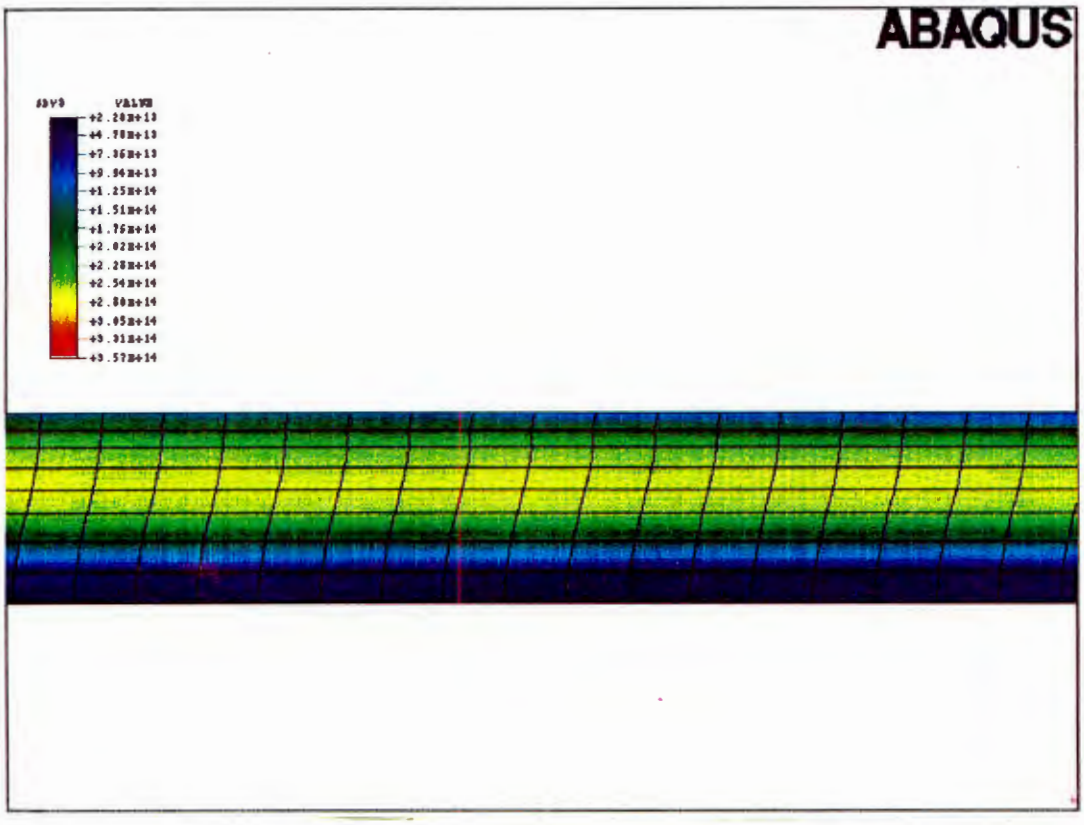


Figure 5.4. Contours of Zener-Hollomon parameter after the third roll pass, showing the banded structure along the rolling direction of the slab.

5.6.2. Average Grain Size

Equation 2.4.8 relates the average grain size (d_{mean}) to the recrystallized and unrecrystallized volume fractions, and their corresponding grain sizes. Figure 5.5 shows the changes in average grain size as a function of time, at three different locations through the thickness of the slab, namely at the surface, at sub-surface (position 0.222 in Figure 5.3), and at the centre. Grain size changes are more prominent at the surface and sub-surface than at the centre, as a result of the lower levels of strain and Zener-Hollomon parameters at the centre.

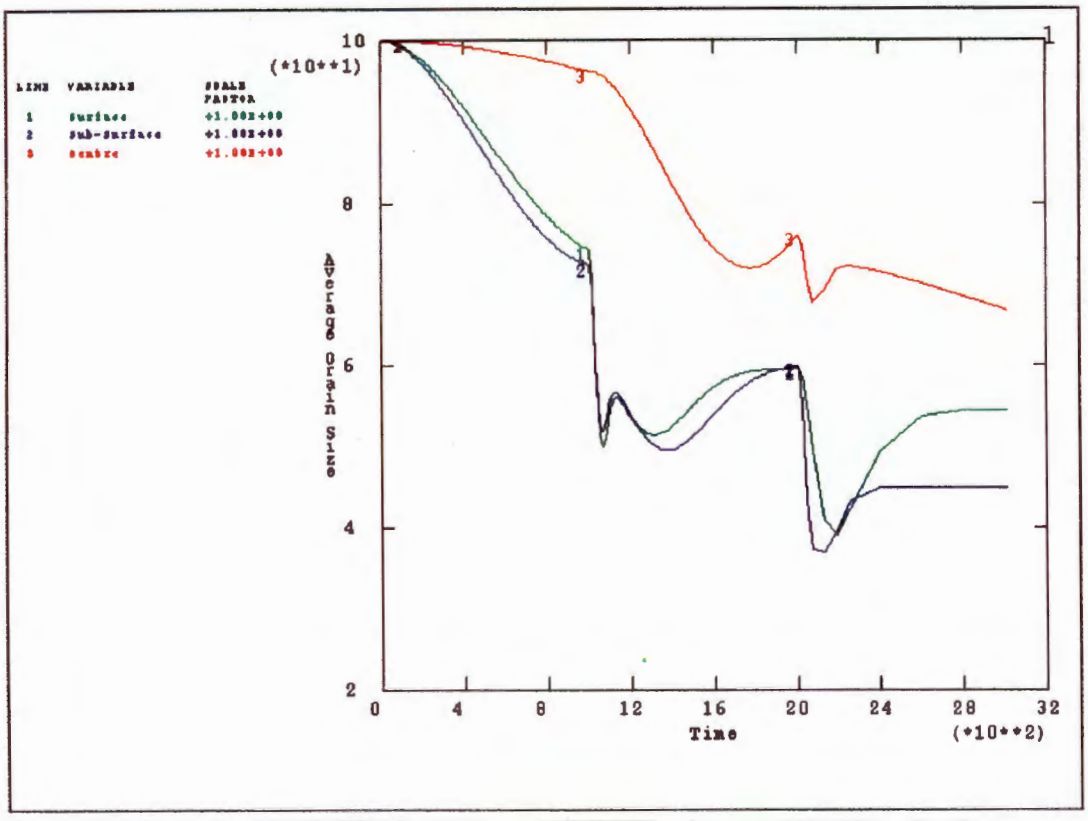


Figure 5.5. Average Grain Size (in microns) versus Time over the entire roll schedule.

5.6.3. Volume Fraction Recrystallized

Figure 5.6 shows the volume fraction recrystallized as a function of time. The recrystallized volume fraction is related to the time for 50% recrystallization by equation 2.4.5, which is in turn related to other parameters described in equation 2.4.3. The behaviour in Figure 5.6 is thus as expected, with the largest amount of recrystallization occurring where strain and Zener-Hollomon parameter values are greatest. Note that each roll pass treats all material as non-recrystallized at the start of each roll pass. Strain, however, is treated in a cumulative fashion, thus explaining the sudden increase in recrystallization rate after the second roll pass at the centre.

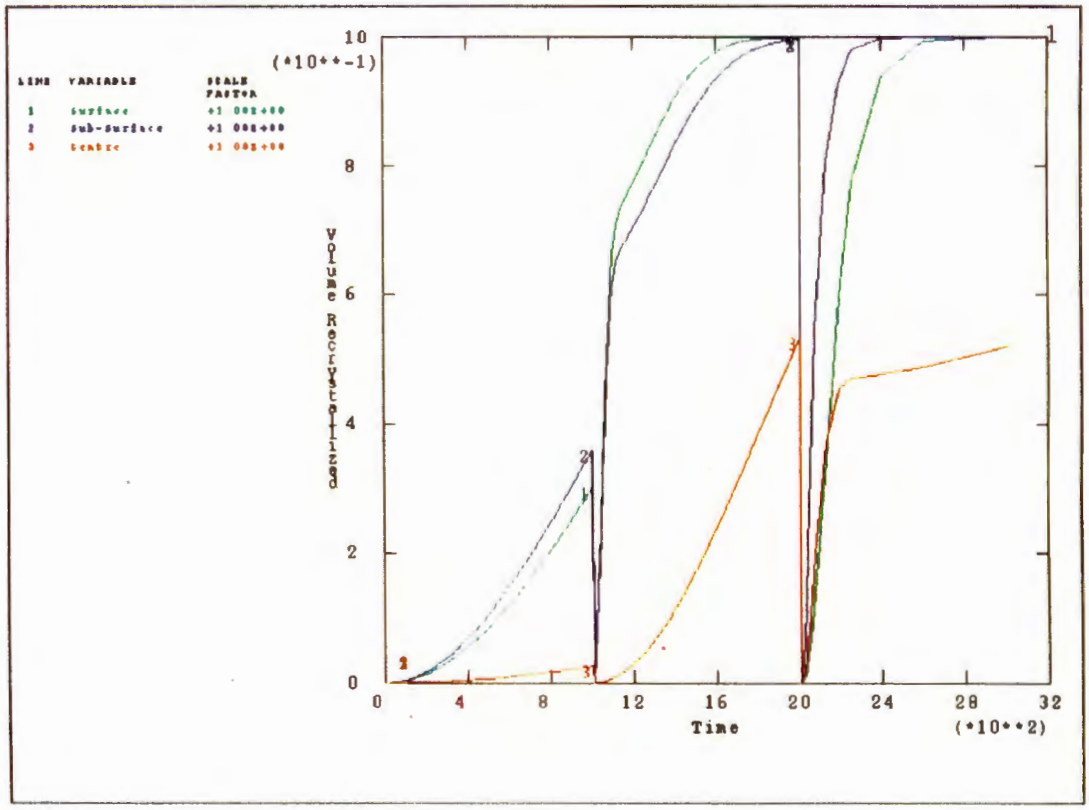


Figure 5.6. Volume Fraction recrystallized as a function of time at three different positions through the thickness of the slab.

5.6.4. Residual Plastic Strain

A decrease in internal plastic strain occurs due to the recovery and recrystallization of the material. This does not imply that the geometrical changes in the material are reversed, it simply indicates that the amount of residual strain in the material is decreasing. Figure 5.7 shows the residual plastic strain changes due to recrystallization over the three roll passes. The surface experiences a greater decrease in residual plastic strain than the centre initially. However, since the strain of the separate roll passes is cumulative, a severe change occurs at the centre after the third roll pass. The behaviour exhibited in Figure 5.7 is confirmed by that in Figure 5.6, where 100% recrystallization at the sub-surface after the third roll pass corresponds to a strain value of zero.

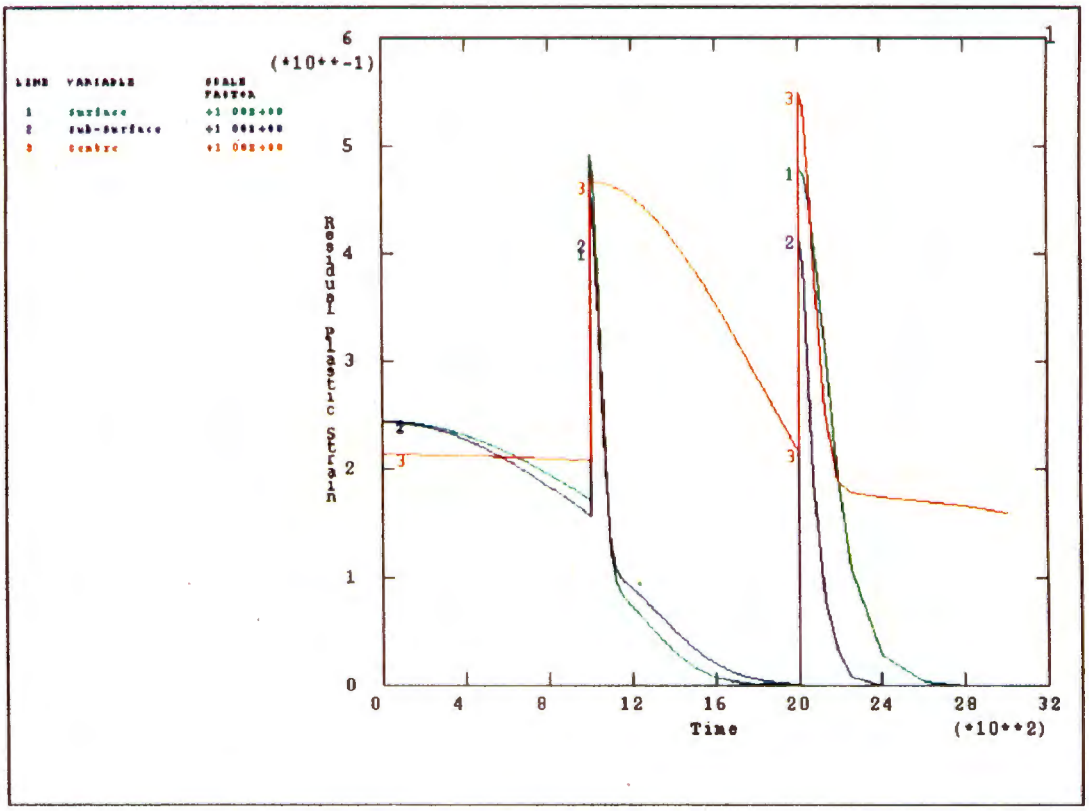


Figure 5.7. Amount of residual plastic strain as a function of time over the three roll passes.

5.7. Summary

The behaviour described by the model in the above figures corresponds with the behaviour which is intuitively expected given the contour plots generated in the analysis. While the test case is hypothetical in nature, it serves to increase the confidence in the model, which is successful in managing all the volume fractions independently. Trends can be investigated, thus making the model suitable for parametric studies, which is described further in the following chapter.

CHAPTER 6

PARAMETRIC STUDY - EFFECT OF ROLL SPEED

6.1. Introduction

Rolling operations involve a large number of variables, and thus parametric studies become necessary in order to analyse a rolling process fully. In Chapter 5 the model was tested in order to check the behaviour of the recrystallization routines. In this chapter, the interpass times are set to more realistic values, and the analyses are repeated using three different roll speeds, namely 150, 180 and 240m/min. These values represent the circumferential speed of the roll, which has a radius of 375mm.

The influence of changing the roll speed influences predominantly the strain rate, and thus the Zener-Hollomon parameter, but since the UMAT has included rate dependency, the amount of plastic strain may also differ slightly. Since the Zener-Hollomon parameter is affected most, this variable has been used as the main tool for comparison.

6.2. Rolling Schedule

The rolling schedule used for this study was similar for all three analyses, with only the roll speed being changed. The schedule is shown in Table 6.1 below.

ROLL PASS	STARTING GAUGE (mm)	FINAL GAUGE (mm)	STANDING TIME (s)
1	60	50	130
2	50	40	200
3	40	30	500

Table 6.1. Rolling Schedule of Test Case

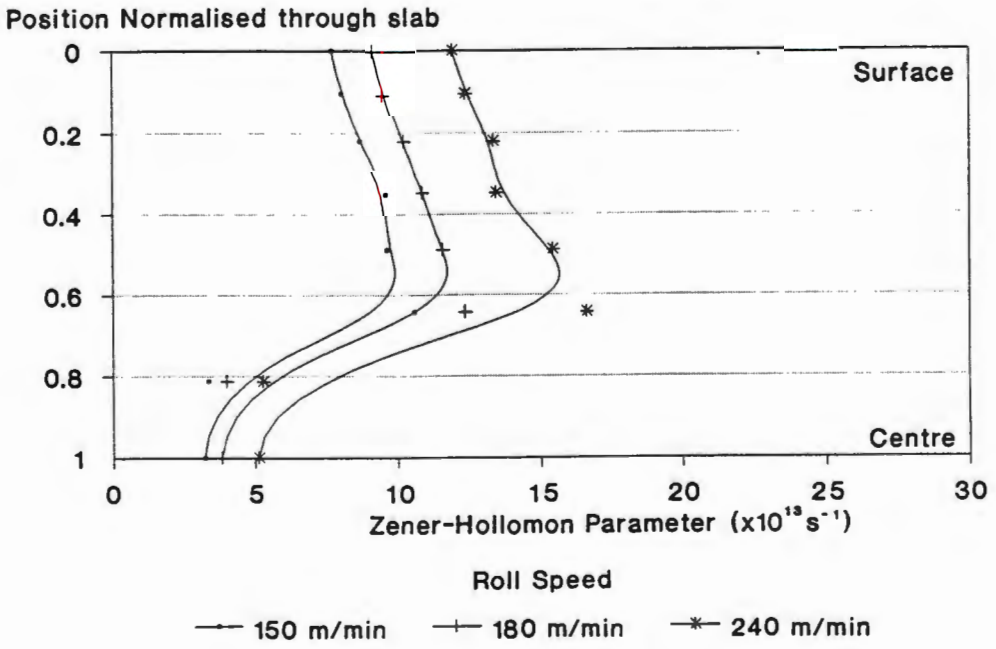
The analyses were performed using a constant temperature of 623K, and the initial grain size was taken as $100 \mu m$.

6.3. Discussion of Results

As in Chapter 5, the average grain size, volume fraction recrystallized, and plastic strain are compared for the entire roll schedule. The Zener-Hollomon parameter is compared through the thickness of the slab, after each roll pass, and compared for the three roll speeds. Each of these comparisons is described in more detail below.

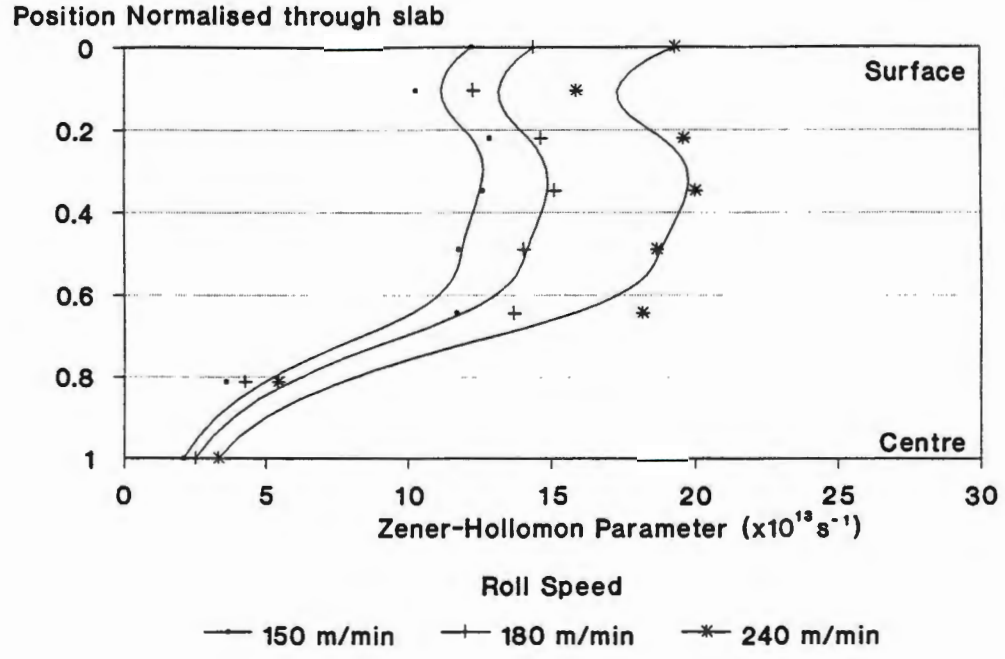
6.3.1. Zener-Hollomon Parameter

The variation of the Zener-Hollomon parameter through the thickness of the slab was compared for each roll pass, showing the difference due to roll speed. Figures 6.1a), b) and c) show this variation, and it can be seen that the Zener-Hollomon parameter, while increasing in magnitude with roll speed, exhibits the same trend through the thickness for each roll pass.

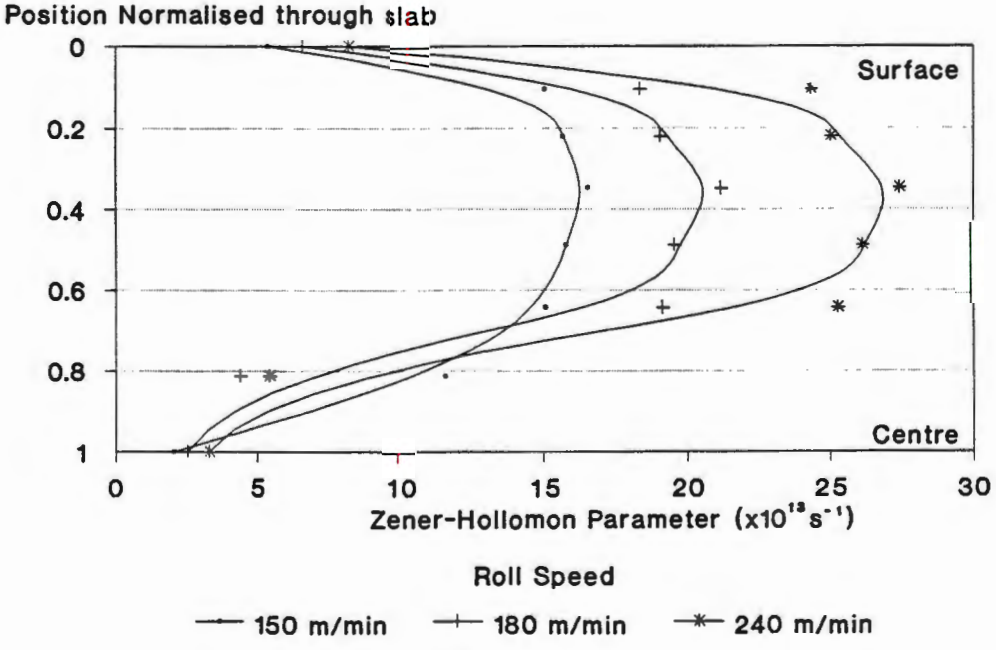


a) Roll Pass 1

Figure 6.1. Variation of Zener-Hollomon parameter through the thickness of the slab at varying roll speeds for a) roll pass 1, b) roll pass 2, and c) roll pass 3.



b) Roll Pass 2



c) Roll Pass 3

Figure 6.1. Continued

Figure 6.2 shows the Zener-Hollomon parameter as a function of roll speed at the surface, sub-surface position, and centre after the third roll pass. The variation in Zener-Hollomon parameter with roll speed is linear at all three positions, with only the slope becoming steeper. The slope is steepest for the sub-surface position, which is the position which varies the most, as can be seen in Figure 6.1.

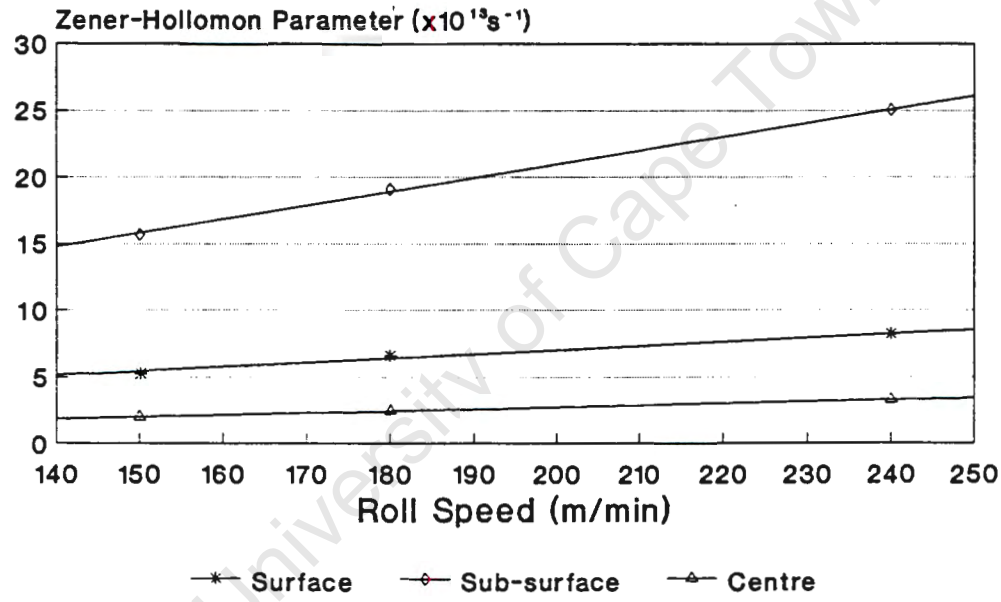


Figure 6.2. Zener-Hollomon parameter as a function of roll speed at different positions after roll pass 3.

The response of Zener-Hollomon parameter at the surface as a function of roll speed is shown in Figure 6.3 for all three roll passes. Once again, the trends are linear with the second roll pass exhibiting the highest values. This can also be seen in Figure 6.1b), where the surface experiences a high strain rate.

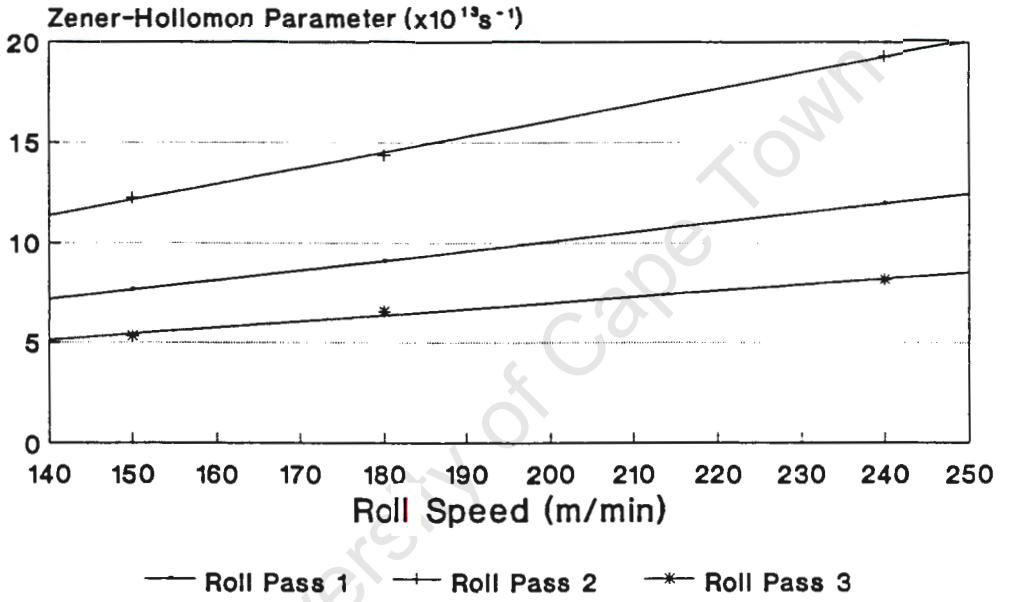


Figure 6.3. Response of Zener-Hollomon parameter at the surface as a function of roll speed after each roll pass.

Sellars and Whiteman [2] relate an average strain rate in rolling to roll speed and roll geometry by using the plane strain conditions which exist during rolling. This allows conversion of rolling strain to von Mises equivalent strain, and the equivalent strain rate is given by

$$\dot{\varepsilon} = \frac{1.155V}{\sqrt{R(h_0 - h_f)}}.\ln\frac{h_0}{h_f} \qquad (6.3.1.)$$

where V is the circumferential speed of the roll in $m.s^{-1}$, R is the radius of the roll, and h_0 and h_f are the original and final gauges, after a roll pass, respectively. Figure 6.4 compares the Sub-surface Zener-Hollomon parameter, as a function of roll speed, with that given by equation 6.3.1. As can be seen, the trends are very close for both roll pass 2 and 3, and only a small mismatch is found in roll pass 1. These results serve to increase confidence in the strain rate measurement technique used in the model, and the model's performance in predicting the Zener-Hollomon parameter is useful for establishing trends which are comparable to calculations used in practice.

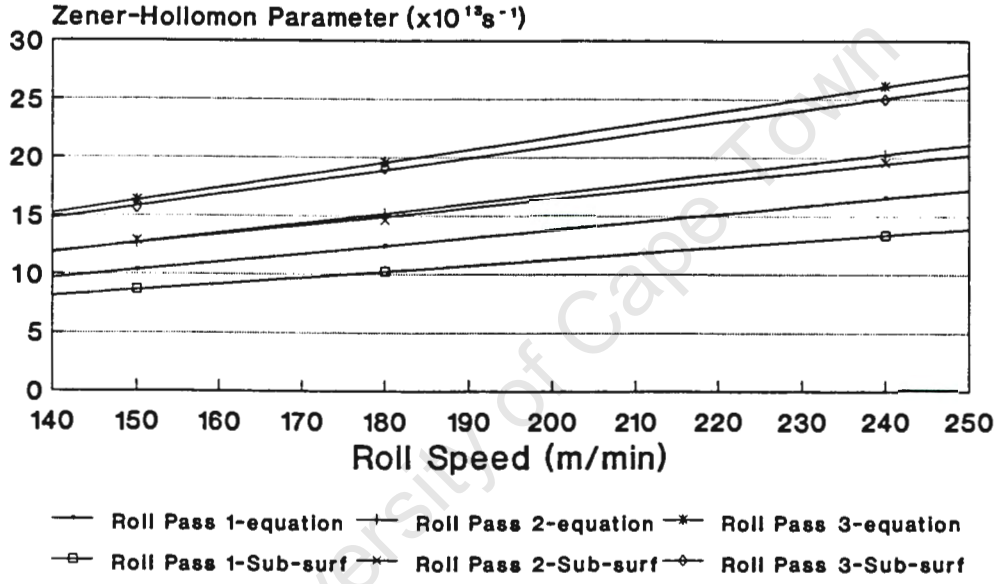
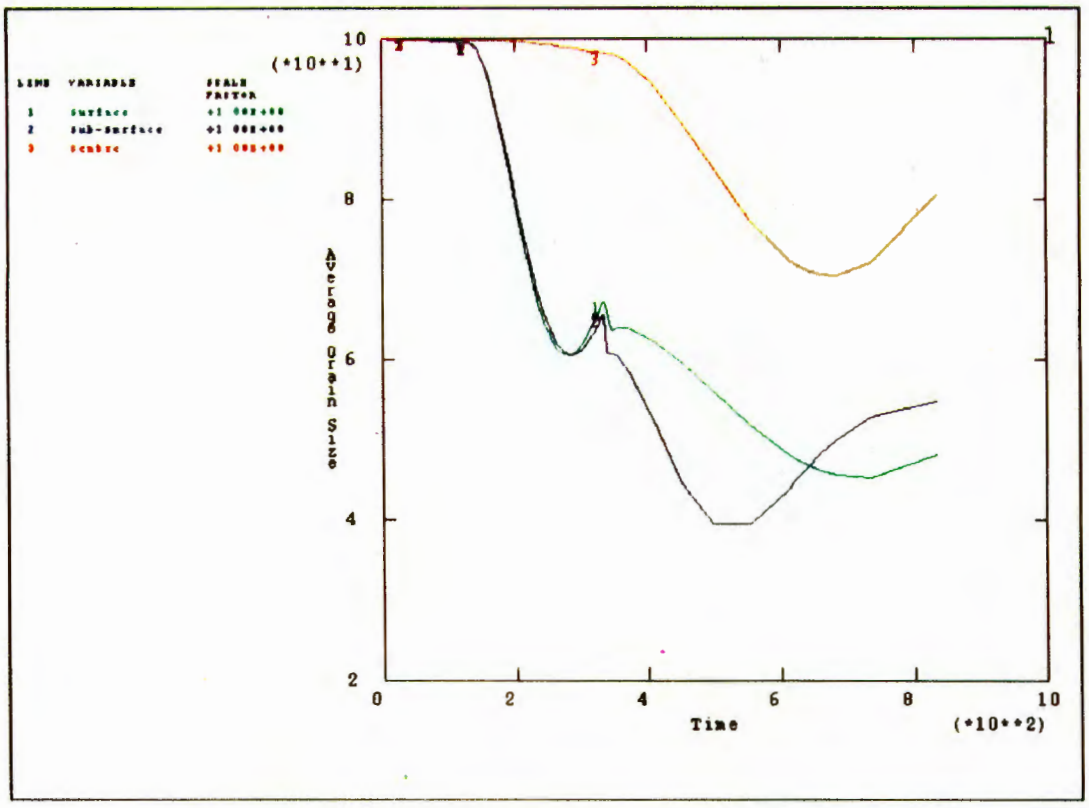


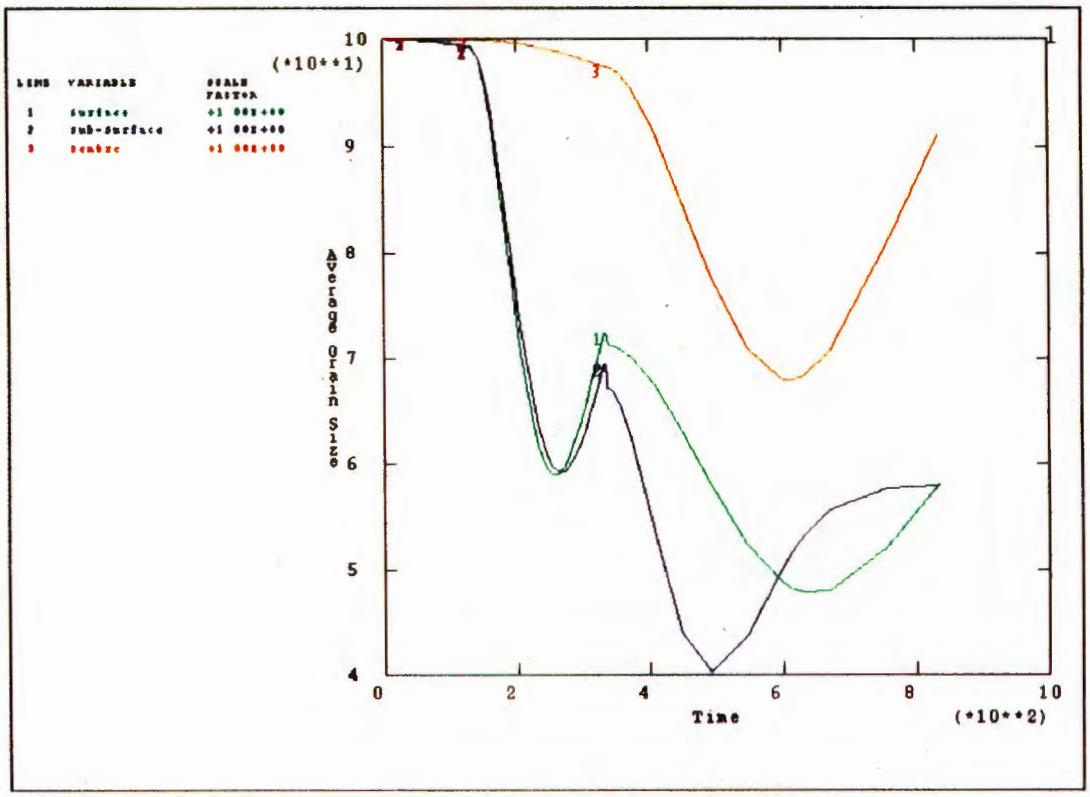
Figure 6.4. Comparison of measured and calculated Zener-Hollomon parameter as a function of roll speed for all three roll passes.

6.3.2. Average Grain Size

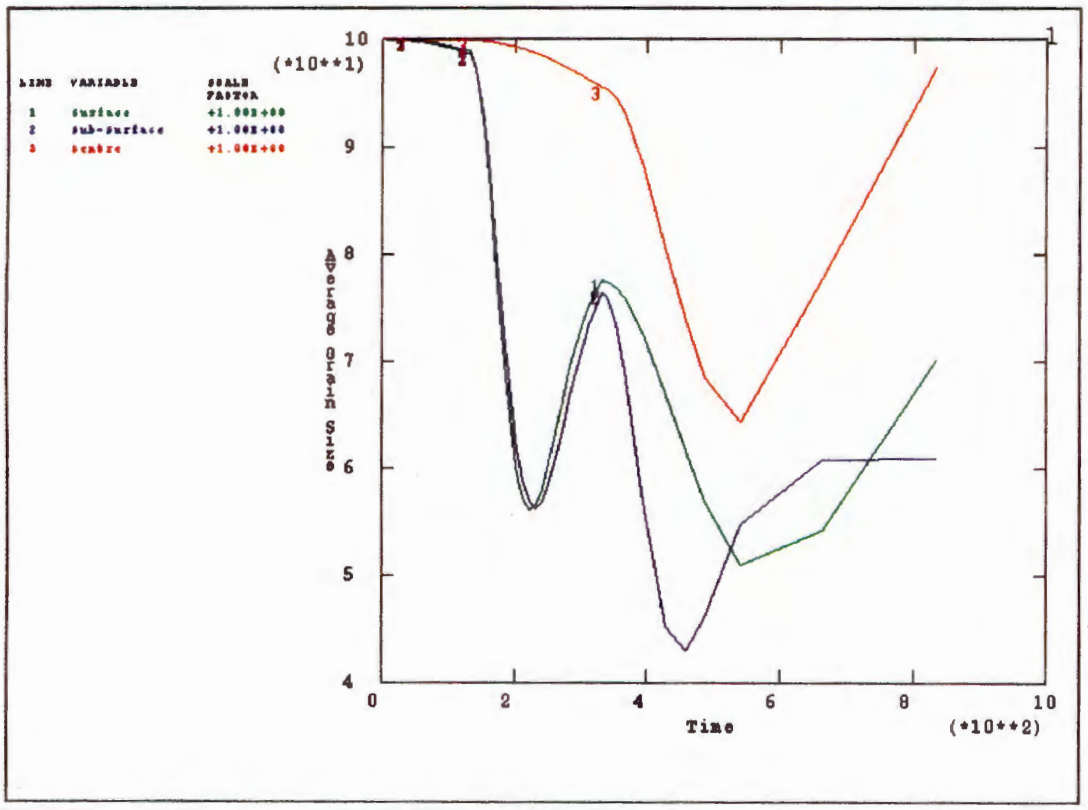
The average grain size (d_{mean}) is plotted as a function of time, for different positions within the slab, for each of the three roll speeds. Figures 6.5a), b) and c) show the evolution of grain size over the rolling time. The grain size changes are most severe for a roll speed of 240m/min, but the basic form of the curves are the same.



a) 150 m/min



b) 180 m/min

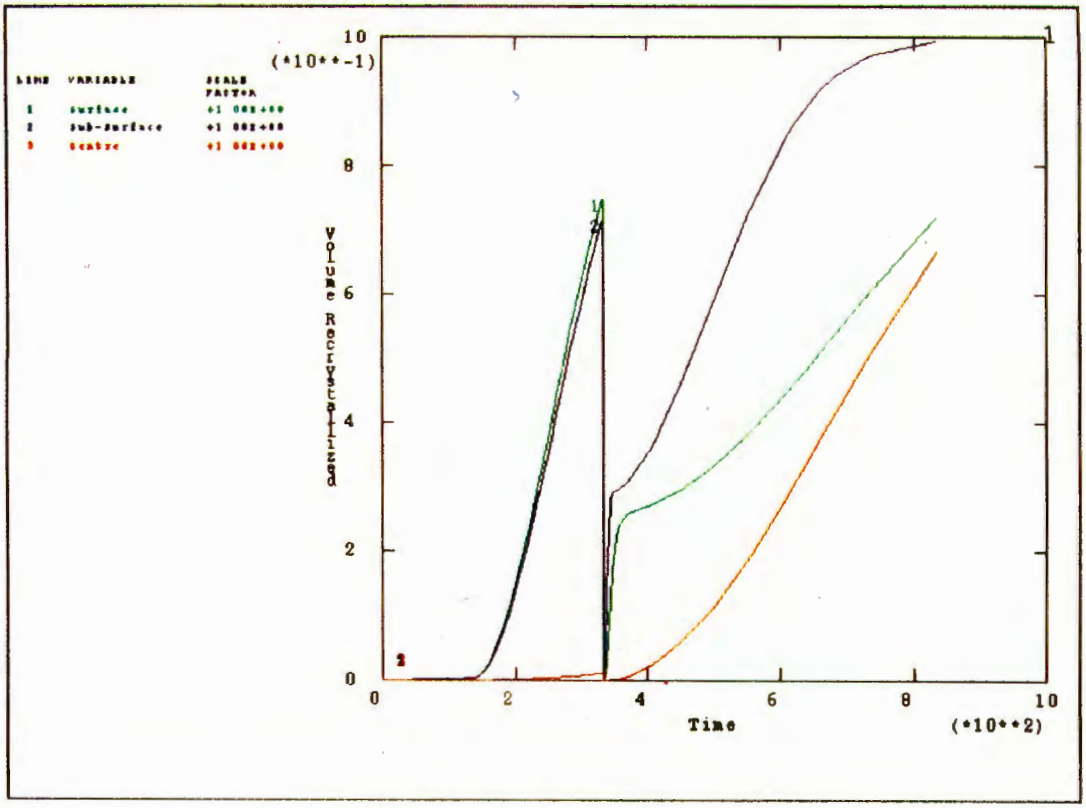


c) 240 m/min

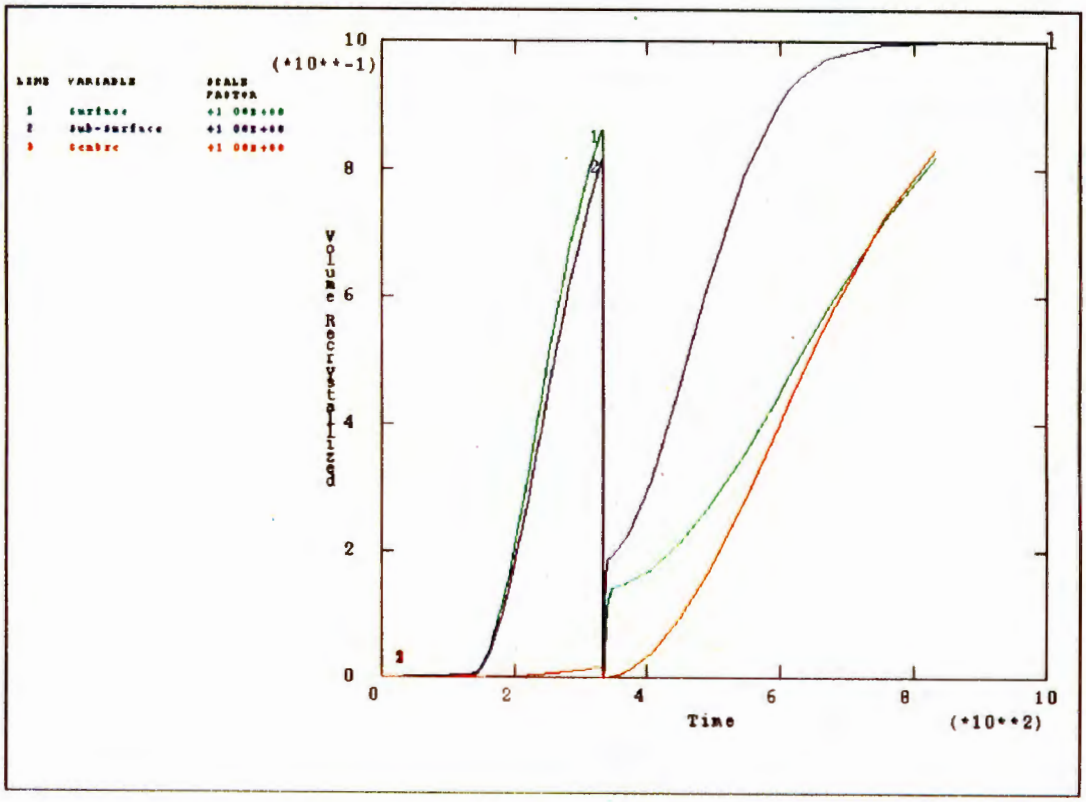
Figure 6.5. Average grain size at different points through the slab for roll speeds of a)150m/min, b)180m/min and c)240m/min.

6.3.3. Volume Fraction Recrystallized

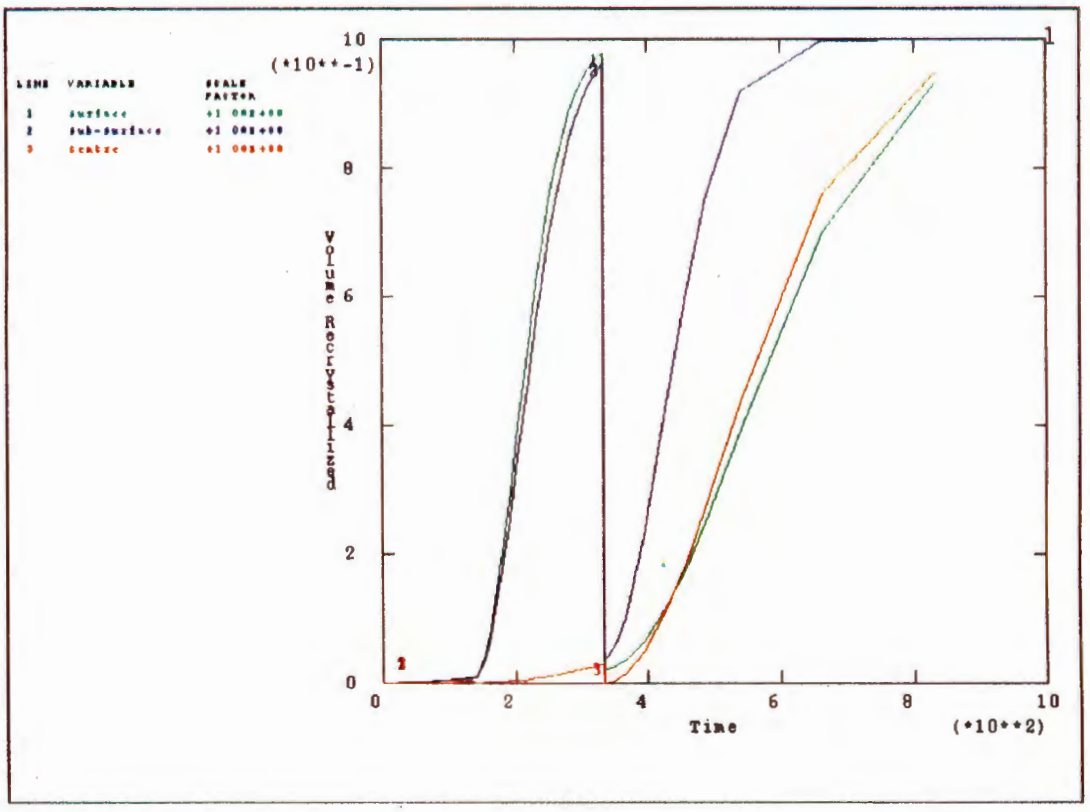
Figures 6.6a), b) and c) show the volume fraction recrystallized versus time for the three different roll speeds. It can be seen that the volume fractions recrystallize faster for the higher roll speeds, since a higher roll speed corresponds with a larger value for the Zener-Hollomon parameter. In all three cases, however, the fastest recrystallization occurs at the sub-surface, with the surface and the centre ultimately recrystallizing at a similar rate.



a) 150 m/min



b) 180 m/min

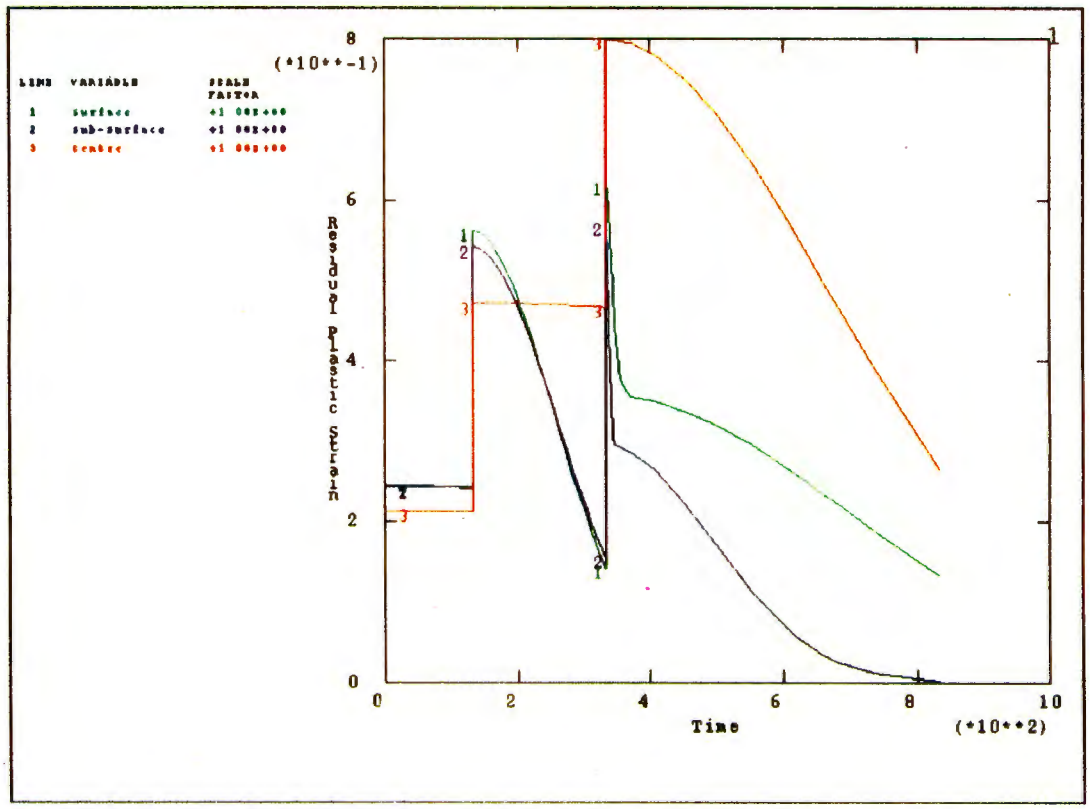


c) 240 m/min

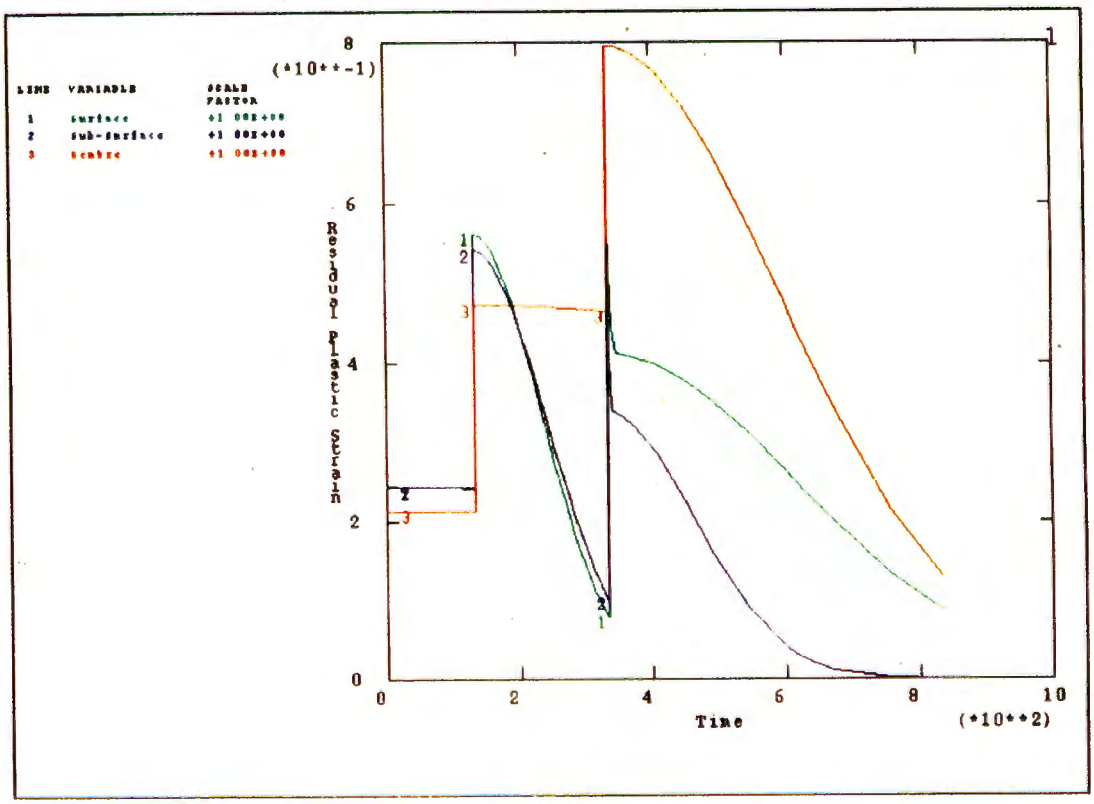
Figure 6.6. Volume Fraction Recrystallized at different points through the slab for roll speeds of a)150m/min, b)180m/min and c)240m/min.

6.3.4. Residual Plastic Strain

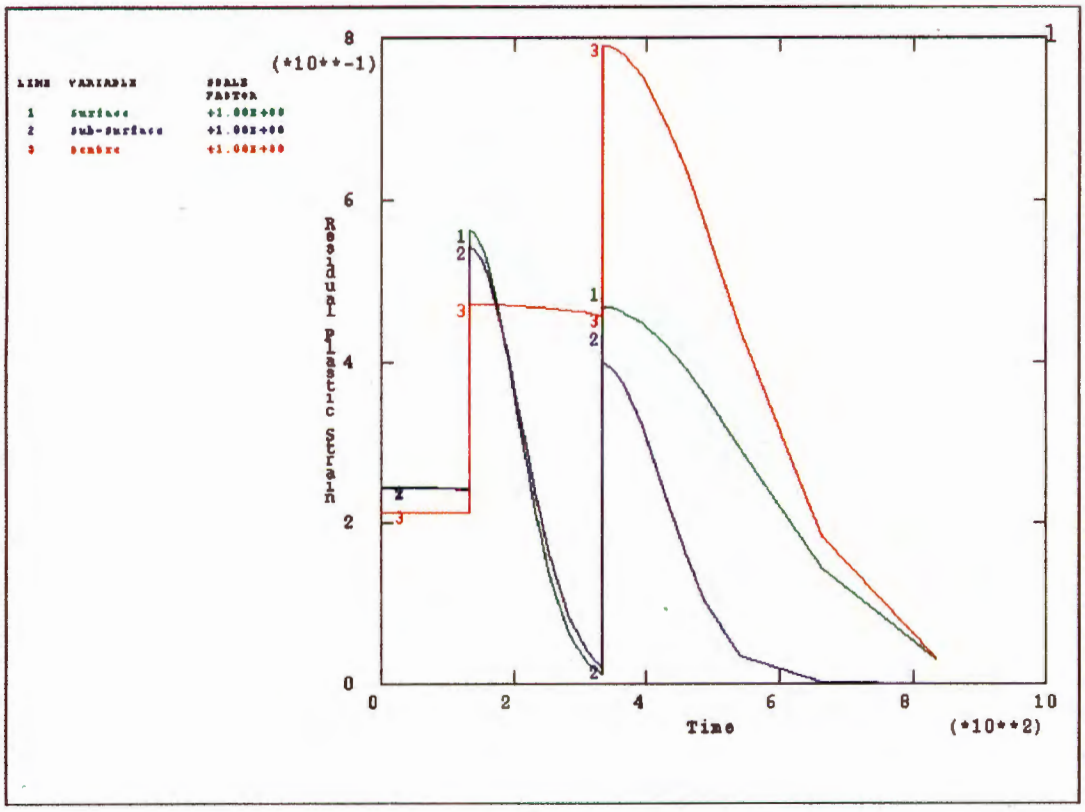
Figure 6.7 shows the residual plastic strain, as a function of time, for all three cases. Once again, the plots for residual plastic strain are in agreement with the plots of volume fraction recrystallized. A strain reduction is most prominent in the analysis using a roll speed of 240m/min. Zero residual strain occurs at the sub-surface, since recrystallization has been completed here. In all three cases, residual strain does not change significantly after the first roll pass, since the strain values are relatively low. Varying amounts of reduction of internal plastic strain occur at the surface and sub-surface positions after the second roll pass, but no significant reduction occurs at the centre, since the strains here are still low. After the third roll pass, however, the cumulative effect of strain, as well as the thinning of the slab, lead to a significant reduction at the centre. This behaviour is exhibited in all three cases, with only the relative amounts being different.



a) 150 m/min



b) 180 m/min



c) 240 m/min

Figure 6.7. Plastic Strain Relaxation at different points through the slab for roll speeds of a)150m/min, b)180m/min and c)240m/min.

6.4. Summary

Three roll speeds were analysed to highlight the effect on the Zener-Hollomon parameter and, in turn, the effect of the Zener-Hollomon parameter on average grain size, volume fraction recrystallized, and residual plastic strain. The trends described in this chapter show that the recrystallization effects are more severe when a faster roll speed is employed. The Zener-Hollomon parameter compares favourably with values calculated using the Sellars and Whiteman equation presented in Section 6.3.1, thus increasing confidence in the model.

CHAPTER 7

CONCLUSIONS AND FUTURE WORK

7.1. Conclusions

The model presented in this thesis has been developed as a starting point for further research into the numerical modelling of multipass rolling operations. While the model has some limitations, it is successful in describing trends in average grain size, volume fraction recrystallized, and residual plastic strain.

The model is further useful in performing parametric studies as was shown in Chapter 6. The results obtained from this study were successful in describing the effect of a single isolated parameter, namely roll speed. In the same way, other parameters such as slab temperature or interpass holding times could be investigated for a specific roll schedule. However, analyses including these parameters will only be meaningful once the heat loss and heat generation in the roll gap have been included in the model.

7.2. Future Work

The future work associated with this model should be to address the problems and limitations which the model currently has. These have been discussed above, but also include grain growth characteristics and the grain size averaging scheme (discussed in Chapter 2).

In addition, experimental validation of the model is still outstanding. An extensive testing program would have to be implemented to obtain accurate material properties for a particular material, as well as simulated rolling tests to describe the recrystallization behaviour of that material. Some of this work is currently underway at the Department of Materials Engineering at the University of Cape Town.

7.2.1. Grain Growth

Grain growth is a phenomena which has not been taken into account in this model, due to the unavailability of data for Al-1%Mg. However, it is an effect which is assumed to occur only once recrystallization is completed, and for the roll schedules used for this thesis, full recrystallization does not occur between roll passes.

In order to include the grain growth characteristics in the model, the recrystallization routines must be updated such that once the volume fraction recrystallized (X_v) is equal to one, grain growth occurs. However, since each of the recrystallizing volume fractions are handled separately, grain growth of a particular component is limited to that volume fraction, which may not be an accurate reflection of the actual situation.

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APPENDIX I

MATERIAL CONSTANTS

A1.3. Recrystallization Constants

Al-1%Mg was used for the purposes of this thesis since material data, including the recrystallization constants, were readily available in literature. Table A1.1 summarises the recrystallization constants used.

EQUATION NUMBER	CONSTANT	DESCRIPTION	VALUE	REF.
2.4.1, 2.4.3	R	Universal Gas Constant	8.314	-
2.4.1	Q_{def}	Activation Energy for Deformation	156000	7,10
2.4.3	α	Material Constant	9.8×10^{-6}	7,10
2.4.3	Q_{rex}	Activation Energy for Recrystallization	230000	7,10
2.4.4	β	Material Constant	435	7,10
2.4.5	n	Material Constant	2	15

TABLE A1.1. Material Constants for Al-1%Mg

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2.4.5	n	Material Constant	2	15

TABLE A1.1. Material Constants for Al-1%Mg

AI Material Constants

AI.1 Static Yield Strength

The value of the yield strength of Al-1%Mg was determined from the strengths of various aluminium alloys containing Mg. These strengths are shown as a function of temperature in Figure A1.1 below. These values suggest that a value between 15MPa and 45MPa for the working temperature, namely 350°C, would be acceptable. A value of 30MPa was used for the analyses.

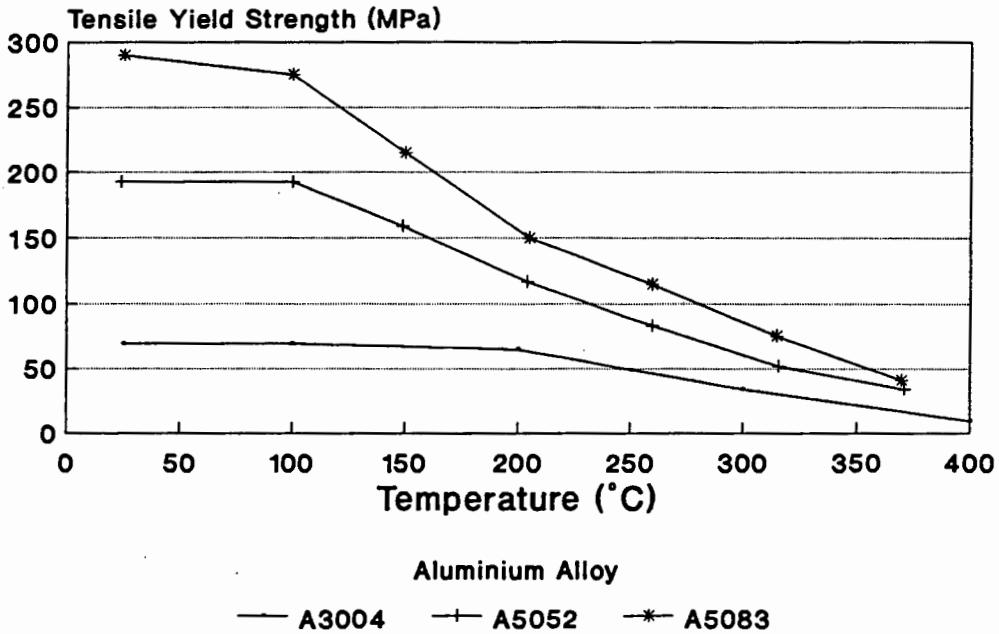

Figure A1.1. Static tensile yield strength versus temperature for three aluminium alloys (Reference 28).

A1.2. Rate Dependent Constants

The temperature dependent constants, D and P, for rate dependent materials were found by rewriting equation 2.3.1 as

$$\frac{\bar{\sigma}}{\sigma_y} - 1 = \left(\frac{\dot{\varepsilon}^{pl}}{D}\right)^{\frac{1}{P}} \qquad \text{(A1.1)}$$

and taking the natural logarithm of both sides gives

$$\ln\left(\frac{\bar{\sigma}}{\sigma_y}-1\right)=\frac{1}{P}\ln\dot{\varepsilon}^{pl}-\frac{1}{P}\ln D \tag{A1.2}$$

Plotting $\ln\left(\frac{\bar{\sigma}}{\sigma_y}-1\right)$ versus $\ln\dot{\varepsilon}^{pl}$ results in a straight line such that

$$P=\frac{1}{slope} \tag{A1.3}$$

and

$$D=e^{-P.const} \tag{A1.4}$$

Perfect plasticity occurs when the elastic strain rate becomes negligible, that is

$$\dot{\varepsilon}^{pl}\approx\dot{\varepsilon}^{tot} \tag{A1.5}$$

Figure A1.2 below, shows the stress-strain curves for Al-1%Mg for different strain rates at 400°C [14]. By plotting these values as described above and performing a linear regression, using a constant static yield stress, the values D and P can be determined. The values determined for D and P this analysis were 1.1466 and 4.8441 respectively.

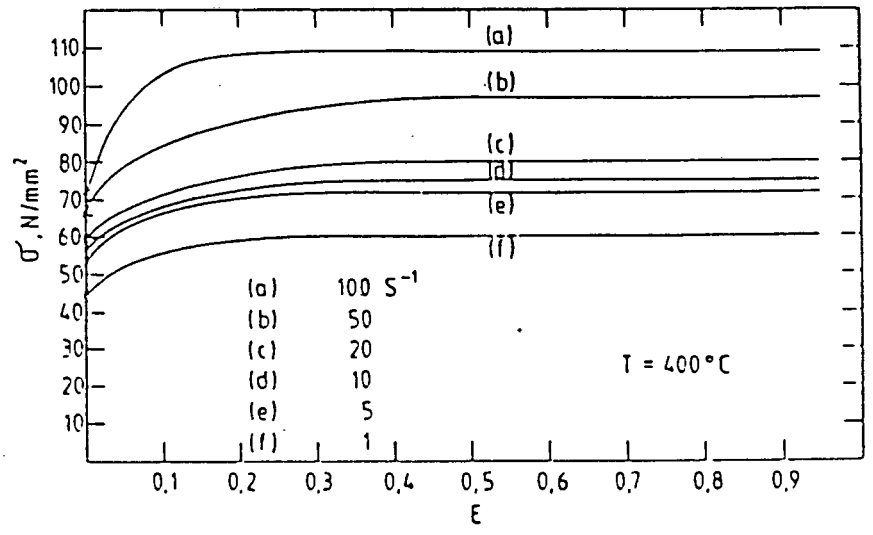

Figure A1.2. Stress strain curves for Al-1%Mg for different strain rates at 400°C (Reference 14).

APPENDIX II

FORTRAN CODING FOR USER SUBROUTINE UMAT INCLUDING RECRYSTALLIZATION ROUTINES

```
      SUBROUTINE UMAT(STRESS,STATEV,DDSDDE,SSE,SPD,SCD,RPL,DDSDDT,
     *           DRPLDE,DRPLDT,STRAN,DSTRAN,TIME,DTIME,TEMP,
     *           DTEMP,PREDEF,DPRED,CMNAME,NDI,NSHR,NTENS,
     *           NSTATV,PROPS,NPROPS,COORDS,DROT,PNEWDT,CELENT)
c.....
c Program..... UMAT
c Purpose..... USER MATERIAL - Isotropic Von Mises plasticity with
c              linear strain hardening and rate dependant hardening.
c Date..... January 1992
c Add software.. RECRYST subroutine
c.....

c     include 'ABA_PARAM.INC'


c.....
c     ABAQUS variables
c.....

c     character*8 CMNAME
c     integer     NDI,                    ! number of direct components
c     *           NSHR,                   ! number of shear components
c     *           NTENS,                  ! direct + shear components
c     *           NSTATV,                 ! number of state variables
c     *           NPROPS                  ! number of properties

c     real*8      STRESS(NTENS),          ! stress vector
c     *           STATEV(NSTATV),         ! state variables
c     *           DDSDDE(NTENS,NTENS),    ! material stiffness matrix
c     *           SSE,                    ! specific elastic strain energy
c     *           SPD,                    ! plastic dissipation
c     *           SCD,                    ! creep dissipation
c     *           RPL,                    ! volumetric heat generation
c     *           DDSDDT(NTENS),          ! variation of stress w.r.t temp
c     *           DRPLDE(NTENS),          ! variation of RPL w.r.t strain
c     *           DRPLDT,                 ! variation of RPL w.r.t temp
c     *           STRAN(NTENS),           ! total strain
c     *           DSTRAN(NTENS),          ! strain increment
c     *           TIME(2),                ! total time at start of inc
c     *           DTIME,                  ! time increment
c     *           TEMP,                   ! temperature
c     *           DTEMP,                  ! temperature increment
c     *           PREDEF(1),              ! predefined state variables
c     *           DPRED(1),               ! increment in PREDEF
c     *           PROPS(NPROPS),          ! user defined properties
c     *           COORDS(3),              ! material point coordinates
c     *           DROT(3,3),              ! rotation increment matrix
c     *           PNEWDT,                 ! suggested new time ratio
c     *           CELENT                  ! characteristic element length

      implicit none
      character*8 CMNAME
      integer     NDI, NSHR, NTENS, NSTATV, NPROPS

      real*8      STRESS(NTENS), STATEV(NSTATV), DDSDDE(NTENS,NTENS),
     *           SSE, SPD, SCD, RPL, DDSDDT(NTENS), DRPLDE(NTENS),
```

```
      *     DRPLDT, STRAN(NTENS), DSTRAN(NTENS), TIME(2), DTIME,
      *     TEMP, DTEMP, PREDEF(1), DPRED(1), PROPS(NPROPS),
      *     COORDS(3), DROT(3,3), PNEWDT, CELENT



c.....
c     internal variables
c.....

c     integer   I,
c     *         J,
c     *         LUN

c     real*8    TTSTRAN(4),          ! total strain
c     *         E_HAT(4),            ! total deviatoric strain
c     *         S(4),                ! deviatoric stress
c     *         OLDSTRES(4),         ! stress in previous increment
c     *         VLSTRAN,             ! volumetric strain
c     *         ELSTRAN(4),          ! elastic strain
c     *         DELSTRAN(4),
c     *         DDSTRAN(4),          ! deviatoric strain increment
c     *         DTSTRAN(4),
c     *         PLSTRAN(4),          ! plastic strain
c     *         DPLSTRAN(4),         ! plastic strain increment
c     *         E_CURL,              ! Mises equivalent strain
c     *         N(4),                ! Plastic flow direction
c     *         SINV1,               ! first stress invariant
c     *         SINV2,               ! second stress invariant
c     *         SIGMA0,              ! static yield strength
c     *         SIGMABR,             ! dynamic yield strength
c     *         DEPL,                ! equivalent plastic strain increment
c     *         B,
c     *         R,
c     *         Q,
c     *         CONVERG,             ! residual
c     *         TOL,                 ! convergence tolerance
c     *         F1,                  ! variables used in secant method
c     *         F2,                  ! variables used in secant method
c     *         H,                   ! static hardening
c     *         HR,                  ! dynamic hardening
c     *         G,                   ! shear modulus
c     *         K,                   ! bulk modulus
c     *         DUMMY,               ! dummy variable
c     *         VSMALL,              ! value approaching zero
c     *         VBIG,                ! value approaching infinity
c     *         ZERO                 ! zero

      integer   I, J, KSTEP

      real*8    TTSTRAN(4), E_HAT(4), S(4), OLDSTRES(4), VLSTRAN,
     &          ELSTRAN(4), DELSTRAN(4), DDSTRAN(4), DTSTRAN(4),
     &          PLSTRAN(4), DPLSTRAN(4), E_CURL, N(4), SINV1, SINV2,
     &          SIGMA0, SIGMABR, DEPL, B, R, Q, CONVERG, TOL,
     &          H, HR, G, K, DUMMY, dummy1, dummy2, VSMALL, VBIG, ZERO
```

```
c.....
c  initialise variables
c.....

      TOL    = 1.D-3
      ZERO   = 0.D0
      VSMALL = 1.D-8
      VBIG   = 1.D6

      SINV1    = ZERO
      SINV2    = ZERO
      VLSTRAN  = ZERO
      E_CURL   = ZERO
      DEPL     = ZERO
      DUMMY1   = ZERO
      DUMMY2   = ZERO


c- a- define state variables

      if (TIME(2).eq.ZERO)then
        statev( 1) = ZERO
        statev( 2) = ZERO
        statev( 3) = ZERO
        statev( 4) = PROPS(9)
        statev( 5) = 1.0D0
        statev( 6) = ZERO
        statev( 7) = ZERO
        statev( 8) = ZERO
        statev( 9) = ZERO
        statev(10) = ZERO
        statev(11) = ZERO
        statev(12) = ZERO
        statev(13) = PROPS(9)
        statev(14) = ZERO
        statev(15) = ZERO
        statev(16) = ZERO
        statev(17) = ZERO
        statev(18) = ZERO
        statev(19) = ZERO
        statev(20) = ZERO
        statev(21) = ZERO
        statev(22) = ZERO
        statev(23) = ZERO
        statev(24) = ZERO
        statev(25) = ZERO
        statev(26) = ZERO
        statev(27) = ZERO
        statev(28) = ZERO
        statev(29) = 1.0D0
        statev(30) = ZERO
        statev(31) = ZERO
```

```
      statev(32) = ZERO
      statev(33) = ZERO
      statev(34) = ZERO
      statev(35) = ZERO
      statev(36) = ZERO
      statev(37) = ZERO
      statev(38) = 653.d0
      statev(39) = 1.0d0
      statev(40) = zero
      statev(41) = 1.0d0
      statev(42) = zero
      statev(43) = 1.0d0
      statev(44) = PROPS(9)
      statev(45) = zero
      statev(46) = zero
      statev(47) = zero
      statev(48) = zero
      statev(49) = zero
      statev(50) = zero
      statev(51) = zero
    endif


c--- set temperature and enable/disable recrystallization for step
    if(time(1).lt.statev(37))then
      statev(39)=statev(39)+1.0d0
      kstep=int(statev(39))

      if(kstep.eq.2)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = zero
      endif

      if(kstep.eq.3)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = 1.0d0
      endif

      if(kstep.eq.4)then
        statev(38) = 623.0d0
        statev(40) = 1.0d0
        statev(41) = zero
      endif

      if(kstep.eq.5)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = zero
      endif

      if(kstep.eq.6)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = 1.0d0
      endif
```

```
      if(kstep.eq.7)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = zero
      endif

      if(kstep.eq.8)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = 1.0d0
      endif

      if(kstep.eq.9)then
        statev(38) = 623.0d0
        statev(40) = 1.0d0
        statev(41) = zero
      endif

      if(kstep.eq.10)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = zero
      endif

      if(kstep.eq.11)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = 1.0d0
      endif

      if(kstep.eq.12)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = zero
      endif

      if(kstep.eq.13)then
        statev(38) = 623.0d0
        statev(40) = zero
        statev(41) = 1.0d0
      endif

      if(kstep.eq.14)then
        statev(38) = 623.0d0
        statev(40) = 1.0d0
        statev(41) = zero
      endif

    endif
    statev(37)=time(1)


c.....
c   calculate material properties
c.....

    G = PROPS(1)/(2.D0*(1.D0 + PROPS(2)))
```

```
      K = PROPS(1)/(3.D0*(1.D0 - 2.D0*PROPS(2)))
      H = (PROPS(5)-PROPS(3))/(PROPS(6) - PROPS(4))
      SIGMA0 = PROPS(3) + H*STATEV(1)



c.....
c   ELASTIC PREDICTOR
c.....

c- 1- record old stress
      do I = 1, NTENS
       OLDSTRES(I) = STRESS(I)
      end do



c- 2- calculate deviatoric strain at beginning of increment
      call DEVIATOR(STRAN,NDI,NTENS,DTSTRAN,DUMMY)



c- 3- calculate deviatoric stress at beginning of increment
      call DEVIATOR(STRESS,NDI,NTENS,S,DUMMY)



c- 4- calculate plastic strain and dev elastic strain at beginning of increment
      do I = 1, NTENS
        if (I .le. NDI) then
          DELSTRAN(I) = S(I)/(2.D0*G)
        else
          DELSTRAN(I) = S(I)/G
        end if
        PLSTRAN(I) = DTSTRAN(I) - DELSTRAN(I)
        if (DABS(PLSTRAN(I)) .lt. VSMALL) then
          PLSTRAN(I) = ZERO
        end if
      end do



c.....
c   calculate Elastic Tangent Stiffness Matrix
c.....

c- 5-
      do I = 1, NTENS
        do J = 1, NTENS
          if ((I.le.NDI).and.(J.le.NDI)) then
            if (I.eq.J) then
              DDSDDE(I,J)=2.D0*G+3.D0*PROPS(2)*K/(1.D0+PROPS(2))
```

```
          else
            DDSDDE(I,J)=3.D0*PROPS(2)*K/(1.D0+PROPS(2))
          end if
        else
          if (I.eq.J) then
            DDSDDE(I,J) = G
          else
            DDSDDE(I,J) = ZERO
          end if
        end if
      end do
    end do
```

```
c.....
c   calculate elastic stresses
c.....

c- 6- calculate total and elastic strain at end of increment
    do I = 1, NTENS
      TTSTRAN(I) = STRAN(I) + DSTRAN(I)
      ELSTRAN(I) = TTSTRAN(I) - PLSTRAN(I)
    end do


c- 7- calculate elastic stresses

    do I = 1, NTENS
      DUMMY = ZERO
      do J = 1, NTENS
        DUMMY = DUMMY + DDSDDE(I,J)*ELSTRAN(J)
      end do
      STRESS(I) = DUMMY
    end do


c- 8- check for ZERO incremental strain

    do I = 1, NTENS
     DUMMY2 = DUMMY2 + DSTRAN(I)**2
    end do


c-8b- go to end if no strain increment

    if (DUMMY2.eq.ZERO) go to 500


c- 9- calculate Von Mises equivalent stress
    call SINV(STRESS,SINV1,SINV2,NDI,NSHR)
```

```
c-10- check for yielding
      if ((SINV2 - SIGMA0) .le. VSMALL) then
        go to 400
      endif


c-11- initialize at first yield
      if (STATEV(41).ne.ZERO.and.STATEV(34).ne.ZERO)then
        call INITCRYS(statev(5),statev(9),statev(13),
     &              statev(17),statev(21),statev(33),statev(34),
     &              statev(35),statev(36),props,nprops,statev(38),
     &              statev(44),statev(48))
      endif


c.....
c   PLASTIC DEFORMATION CALCULATIONS
c.....

c-12- calculate dev elastic strain + increment in dev strain
      call DEVIATOR(DSTRAN,NDI,NTENS,DDSTRAN,DUMMY)



c-13- calculate E_HAT
      do I = 1, NTENS
        E_HAT(I) = DELSTRAN(I) + DDSTRAN(I)
      end do



c-14- calculate E_CURL (sqr(3/2 E_HAT:E_HAT))
      DUMMY = ZERO
      do i=1, NTENS
        if (I.le.NDI ) then
          DUMMY = DUMMY + E_HAT(I)**2
        else
          DUMMY = DUMMY + 2.D0*(E_HAT(I)/2.D0)**2
        end if
      end do
      E_CURL = sqrt(3.D0*DUMMY/2.D0)



c.....
c   evaluate DEPL using newtons method
c.....

c-15- Newton Loop

      I     = 0
      DEPL  = (SINV2-PROPS(3)-H*STATEV(1))/
     &       (3.D0*G+H+((1.D0/(DTIME*PROPS(7)))**(1.D0/PROPS(8)))
     &       *PROPS(1)**(1.D0)*(H*STATEV(1)+H+1.D0))

100   continue
```

```
      I = I + 1
      if (I.ge.50) then
        stop 'Excessive Newton Iterations Required'
      end if


      DUMMY = DEPL
      DEPL = DEPL + (SINV2-3.D0*G*DEPL-( PROPS(3)+H*(STATEV(1)+DEPL) )*
     &     ( 1.D0+( DEPL/(DTIME*PROPS(7)) )**( 1.D0/PROPS(8)) )) /
     &     (3.D0*G+H*(1.D0+(DEPL/(DTIME*PROPS(7)))**(1.D0/PROPS(8)) )
     &     + ( PROPS(3)+H*(STATEV(1)+DEPL) )*
     &     (1.D0/(DTIME*PROPS(7)*PROPS(8)))*(DEPL/(DTIME*PROPS(7)))**
     &      ( 1.D0/PROPS(8) - 1.D0) )

      if (DEPL.le.ZERO) then
        DEPL = DUMMY/1.D1
        go to 100
      end if

      SIGMABR = SIGMA0*(1.D0+(DEPL/(DTIME*PROPS(7)))**(1.D0/PROPS(8)))

      CONVERG = DABS(SINV2 - 3.D0*G*DEPL - SIGMABR)
      if (CONVERG.ge.(TOL*SIGMABR)) go to 100

200   continue


c-16- update state variables unless not real step
      if(statev(41).ne.zero)then
        STATEV( 1) = STATEV( 1) + DEPL
        STATEV(21) = STATEV(21) + DEPL
        STATEV(22) = STATEV(22) + DEPL
        STATEV(23) = STATEV(23) + DEPL
        STATEV(24) = STATEV(24) + DEPL
        STATEV(35) = STATEV(35) + DTIME
        STATEV(36) = STATEV(36) + DEPL
        STATEV( 2) = STATEV(36)/STATEV(35)
      endif



c-17- calculate new deviatoric stress ( S )
      do I = 1, NTENS
        if (I.le.NDI) then
          S(I) = 2.D0*G*E_HAT(I)/(1.D0+3.D0*G*DEPL/SIGMABR)
        else
          S(I) = G*E_HAT(I)/(1.D0+3.D0*G*DEPL/SIGMABR)
        end if
      end do



c-18- calculate volumetric strain
      VLSTRAN = ZERO
      do I = 1, NDI
        VLSTRAN = VLSTRAN + TTSTRAN(I)
```

```
      end do


c-19- update stress to new state
      do I = 1, NTENS
        if (I.le.NDI) then
          STRESS(I) = S(I) + K*VLSTRAN
        else
          STRESS(I) = S(I)
        end if
      end do




c.....
c    calculate Plastic Tangent Stiffness Matrix
c.....

      HR =  H*(1.D0+(DEPL/(DTIME*PROPS(7)))**(1.D0/PROPS(8)))
     &      + (PROPS(3)+H*(STATEV(1)+DEPL))*
     &     (1.D0/(DTIME*PROPS(7)*PROPS(8)))*(DEPL/(DTIME*PROPS(7)))**
     &       ( 1.D0/PROPS(8) - 1.D0)


c-20a calculate constants
      B = HR/(3.0D0*G)
      Q = SIGMABR/E_CURL
      R = 3.D0*(1.D0-DEPL*HR/SIGMABR)/(2.D0*SIGMABR*E_CURL*(1.0D0+B))



c-20b calculate matrix
      do I = 1, NTENS
        do J = 1, NTENS
          if ((I.le.NDI).and.(J.le.NDI)) then
            if (I.eq.J) then
              DDSDDE(I,J) = K + 2.D0*Q/3.D0 - R*S(I)*S(J)
            else
              DDSDDE(I,J) = K - Q/3.D0 - R*S(I)*S(J)
            end if
          else
            if (I.eq.J) then
              DDSDDE(I,J) = Q/2.D0 - R*S(I)*S(J)
            else
              DDSDDE(I,J) = - R*S(I)*S(J)
            end if
          end if

c-20c check for small values
          if (DABS(DDSDDE(I,J)) .le. VSMALL) then
            DDSDDE(I,J) = ZERO
          end if
        end do
      end do
```

```
c-21- calculate flow stress normal
      do I = 1, NTENS
        N(I) = 3.D0*S(I)/(SIGMABR*2.D0)
      end do


c-23- calculate incremental plastic strain
      do I = 1, NTENS
        DPLSTRAN(I) = DEPL * N(I)
      end do


c-24- calculate total elastic strain
      do I = 1, NTENS
        if (I.le.NDI) then
          ELSTRAN(I) = TTSTRAN(I) - PLSTRAN(I) - DPLSTRAN(I)
        else
          ELSTRAN(I) = TTSTRAN(I)-PLSTRAN(I) - 2.D0*DPLSTRAN(I)
        end if
      end do


c-26- calculate dissipated plastic energy

      DUMMY = ZERO
      do I = 1, NTENS
        DUMMY = DUMMY + DPLSTRAN(I) * (STRESS(I) + OLDSTRES(I))/2.D0
      end do
      SPD = SPD + DUMMY


400   continue

c-27- calculate elastic specific strain energy

      DUMMY = ZERO
      do I = 1, NTENS
         DUMMY = DUMMY + STRESS(I)*ELSTRAN(I)/2.D0
      end do
      SSE = DUMMY


500   continue

c-28- call to recryst
      if (statev(40).ne.zero)then

c--- calculate recrystallization and grain size
```

```
      call CRYSTAL(statev(1),statev(2),statev(3),statev(4),statev(5),
     &          statev(9),statev(13),statev(17),statev(21),statev(25),
     &          statev(29),statev(33),statev(34),dtime,statev(38),
     &          statev(42),statev(43),statev(44),statev(48))

      end if


c-29- return
      RETURN
      END




      subroutine DEVIATOR(VECTOR,NDI,NTENS,DVECTOR,TRACE)
c:.....
c     Purpose:
c     This subroutine calculates and returns the deviatoric tensor
c     from the original tensor.
c     Description of parameters:
c     i vector  incoming tensor
c     i ndi     number of direct components
c     i ntens   totlal number of components
c     o dvector resultant deviatoric tensor
c     o trace   trace of vector
c.....

      implicit   NONE

      integer NDI, NTENS, i
      real*8 TRACE, VECTOR(NTENS), DVECTOR(NTENS)


      TRACE = 0.D0

c.....
c     calculate trace of tensor
c.....

      do 10, I = 1, NDI
        TRACE = TRACE + VECTOR(I)
10    continuE



c.....
c     calculate deviatoric components
c.....

      do 20, I = 1, NTENS
        if( I .le. NDI) then
          DVECTOR(I) = VECTOR(I) - TRACE / 3.D0
        else
```

```
          DVECTOR(I) = VECTOR(I)
        end if
20    continue

      return
      end



      subroutine CRYSTAL(epl,epldot,zener,dmean,x0,xv,d0,drex,
     &                   strain,thalf,ratio,ttime,check,dtime,
     &                   temp,rexvol,unrexv,ds,dr)
c.....................................................................
c
c    Purpose:
c     To calculate recrystallized grain size
c
c    Parameters:                                          State Variable #
c io  epl       - plastic strain from ABAQUS                    1
c i   epldot    - plastic strain rate from ABAQUS               2
c  o  zener     - temperature compensated strain rate           3
c  o  dmean     - mean grain size (diameter)                    4
c i   x0(i)     - array of i initial vol fracs                 5-8
c  o  xv(i)     - array of i recrystallized vol fracs         9-12
c i   d0(i)     - array of i initial grain sizes             13-16
c  o  drex(i)   - array of i recrystallized grain sizes      17-20
c  o  strain(i) - array of i strains modified by rex         21-24
c  o  thalf(i)  - array of i times to 50% rex                25-28
c int ratio(i)  - array of i ratio values to calc vol fracs  29-32
c int ttime     - total time during recrystallization           33
c io  check     - variable used to check deformation status      34
c  o  stime     - variable to time the deformation              35
c  o  sstrain   - total plastic strain during 1 def. cycle      36
c ext steptime  - variable to keep step time                    37
c io  temp      - temperature                                   38
c ext step#     - step number                                   39
c ext  -        - variable used to able/disable CRYSTAL         40
c ext  -        - variable used to able/disable INITCRYS        41
c  o  rexvol    - total vol frac recrystallized                 42
c  o  unrexv    - total vol frac unrecrystallized               43
c  o  ds(i)     - current sacrificial grain size            44-47
c  o  dr(i)     - current recrystallizing grain size        48-51
c
c  where
c     i  - input
c     o  - output
c    int - used for internal calculations
c    ext - used in main UMAT, but not in recrystallization
c
c
c  Other Internal Variables:
c     r0       - zero
c     r        - universal gas constant
c     alfa     - material constant
```

```
c    beta     - material constant
c    n        - material constant
c    total    - variable used for normalizing after small vol
c               constraints have been applied
c    q1       - material constant - energy for zener
c    q2       - material constant - energy for thalf
c
c
c    NOTE: Program specific to three roll passes which result in a
c          maximum of 4 initial grain sizes and 4 recrystallized
c          grain sizes.  Thus parameter NUMVOL is set to 4 and the
c          arrays which are stored within the STATEV array are dim-
c          ensioned to this value.  To apply to more rollpasses, the
c          number of state variables must be increased and adjusted,
c          and NUMVOL must be increased to 2**(x-1) where x is the
c          number of rollpasses
c.....................................................................

      implicit none
      integer numvol,i
      parameter(numvol=4)
      real*8 epl,epldot,zener,dmean,x0(numvol),xv(numvol),d0(numvol),
     &       drex(numvol),strain(numvol),thalf(numvol),dtime,temp,ttime,
     &       ratio(numvol),r0,r,alfa,beta,n,q1,q2,check,rexvol,unrexv,
     &       ds(numvol),dr(numvol)

c--- define variables
      r0    = 0.0d0
      r     = 8.314d0
      alfa  = 9.8d-6
      beta  = 435.0d0
      n     = 2.0d0
      q1    = 156000.d0
      q2    = 230000.d0
      check = 1.0d0
      rexvol = r0
      unrexv = r0

c- 1- set start ratios for calculation
      if(ttime.eq.r0)then
        do 10 i=1,numvol
          ratio(i)=x0(i)
 10     continue
      endif


c---  start time counting
      ttime=ttime+dtime


c- 2- calculate strain rate and Zener-Hollomann parameter and set strain to zero
      zener = epldot*exp(q1/(r*temp))
      epl   = r0
      dmean = r0


c- 3- calculate strain and recrystallized grain size for each vol. fraction
      do 20 i = 1,numvol
```

```
      thalf(i)=alfa*(d0(i)**1.35d0)*(strain(i)**(-2.7d0))*
     &          (zener**(-1.1d0))*exp(q2/(r*temp))

      drex(i)=beta*(d0(i)**1.3d0)*(strain(i)**(-0.39d0))*
     &          (zener**(-0.24d0))


c--- calculate volume fractions of recrystallized and deformed material
c    and apply volume fraction constraints
      xv(i)=1-exp(-0.693d0*((ttime/thalf(i))**n))

c--- calculate recrystallizing and sacrificial grain sizes
      dr(i)=(xv(i)**(1.0d0/3.0d0))*drex(i)
      ds(i)=(1.0d0-xv(i))*d0(i)


c--- calculate average grain size volume fraction
      xv(i)=ratio(i)*xv(i)
      x0(i)=ratio(i)-xv(i)

c--- calculate mean grain size
      dmean=dmean+xv(i)*dr(i)+x0(i)*ds(i)
c       dmean=dmean+(xv(i)**(4.d0/3.d0))*drex(i)+
c    &        (x0(i)**(4.0d0/3.0d0))*d0(i)

c--- calculate new residual strain
      epl=epl+x0(i)*strain(i)

c--- calculate total volume fractions
      rexvol=rexvol+xv(i)
      unrexv=1.0d0-rexvol

 20   continue

c--- calculate mean grain size over entire roll time
c     dmean=dmean-(x0(1)**(4.0d0/3.0d0))*d0(1)
c     dmean=dmean+(x0(1)**2)*d0(1)

 40   continue
      return
      end



      subroutine INITCRYS(x0,xv,d0,drex,strain,ttime,check,stime,
     &                    sstrain,props,nprops,temp,ds,dr)
c.....................................................................
c
c    Purpose:
c     To calculate recrystallized grain size
c
c    Parameters:                                    State Variable #
c io  epl      - plastic strain from ABAQUS                1
c i   epldot   - plastic strain rate from ABAQUS           2
c  o  zener    - temperature compensated strain rate       3
```

```
c o dmean    - mean grain size (diameter)                4
c i x0(i)    - array of i initial vol fracs              5-8
c o xv(i)    - array of i recrystallized vol fracs       9-12
c i d0(i)    - array of i initial grain sizes            13-16
c o drex(i)  - array of i recrystallized grain sizes     17-20
c o strain(i) - array of i strains modified by rex       21-24
c o thalf(i) - array of i times to 50% rex               25-28
c int ratio(i) - array of i ratio values to calc vol fracs 29-32
c int ttime  - total time during recrystallization       33
c io check   - variable used to check deformation status 34
c o stime    - variable to time the deformation          35
c o sstrain  - total plastic strain during 1 def. cycle  36
c ext steptime - variable to keep step time              37
c io temp    - temperature                               38
c ext step#  - step number                               39
c ext -      - variable used to able/disable CRYSTAL     40
c ext -      - variable used to able/disable INITCRYS    41
c o rexvol   - total vol frac recrystallized             42
c o unrexv   - total vol frac unrecrystallized           43
c o ds(i)    - current sacrificial grain size            44-47
c o dr(i)    - current recrystallizing grain size        48-51
c
c where
c   i  - input
c   o  - output
c   int - used for internal calculations
c   ext - used in main UMAT, but not in recrystallization
c
c
c Other Internal Variables:
c   r0      - zero
c   r       - universal gas constant
c   alfa    - material constant
c   beta    - material constant
c   n       - material constant
c   total   - variable used for normalizing after small vol
c             constraints have been applied
c   q1      - material constant - energy for zener
c   q2      - material constant - energy for thalf
c
c
c   NOTE: Program specific to three roll passes which result in a
c       maximum of 4 initial grain sizes and 4 recrystallized
c       grain sizes.  Thus parameter NUMVOL is set to 4 and the
c       arrays which are stored within the STATEV array are dim-
c       ensioned to this value.  To apply to more rollpasses, the
c       number of state variables must be increased and adjusted,
c       and NUMVOL must be increased to 2**(x-1) where x is the
c       number of rollpasses
c.....................................................................
      implicit none
      integer numvol,i,nprops
      parameter(numvol=4)
      real*8 x0(numvol),xv(numvol),strain(numvol),d0(numvol),
     &     drex(numvol),ttime,r0,total,check,stime,sstrain,count,
     &     props(nprops),temp,ds(numvol),dr(numvol)

c--- define variables
```

```
      r0     = 0.0d0
      total  = r0

c---  update variables for recrystallization routines
      check  = r0
      ttime  = r0
      stime  = r0
      sstrain = r0
      count = count+1.d0

c---  convert to bulk volume fractions
      do 10 i=1,numvol

c---  activate total vol. frac. to normalize
       total=total+x0(i)+xv(i)

c---  end calculation
 10   continue

c---  pack data for next rollpass and normalize
      call COMPRESS(numvol,x0,xv,d0,drex,strain,ds,dr)
      do 20 i=1,numvol
        x0(i)=x0(i)/total
 20   continue

      return
      end


      subroutine COMPRESS(numvol,x0,xv,d0,drex,strain,ds,dr)
c.....................................................................
c     Purpose:
c       To reorganize the data (strain, volume fractions, grain sizes, etc.)
c       into a linked arrays, discarding all zero values.  This sets up the
c       environment for the next roll pass.
c.....................................................................

c---  declare variables
      integer numvol, kount0, kountv
      real*8  r0
      parameter(r0=0.0d0)
      real*8  x0(numvol), xv(numvol), d0(numvol), drex(numvol)
      real*8  strain(numvol), ds(numvol), dr(numvol)


c---  initialize counters
      kount0=0
      kountv=0

c---  pack data within array x0 and keep count of number of zero values
      do 30 i=numvol,1,-1
        if(x0(i).eq.0.0)then
          do 40 j=i,numvol-1
            x0(j)=x0(j+1)
            strain(j)=strain(j+1)
            ds(j)=ds(j+1)
            x0(numvol)=r0
 40       continue
```

```
        kount0=kount0+1
       endif
 30   continue

c--- pack data within array xv and keep count of number of zero values
      do 50 i=numvol,1,-1
       if(xv(i).eq.r0)then
        do 60 j=i,numvol-1
         xv(j)=xv(j+1)
         dr(j)=dr(j+1)
         xv(numvol)=r0
 60      continue
        kountv=kountv+1
       endif
 50   continue

c--- redefine counters to reflect number of non-zero values in each array
      kount0=numvol-kount0
      kountv=numvol-kountv

c--- pack data for new roll pass
      do 70 i=kount0+1,kount0+kountv
       x0(i)=xv(i-kount0)
       ds(i)=dr(i-kount0)
       strain(i)=r0
 70   continue

c--- redefine the original grain size array
      do 80 i=1,numvol
       d0(i)=ds(i)
 80   continue

c--- set all values in arrays xv, drex, ds and dr to zero
      call ZERO(xv,numvol)
      call ZERO(drex,numvol)
      call ZERO(ds,numvol)
      call ZERO(dr,numvol)

      return
      end



      subroutine ZERO(array,n)
c.....
c     Purpose:
c       To set all values within an array to zero
c  i -  array:        array to be set to zero
c  i -  n:            size of array
c.....

c--- declare variables
      integer n
      real*8  r0
      parameter(r0=0.0d0)
      real*8  array(n)

c--- set all values to zero
```

```
      do 10 i=1,n
        array(i)=r0
10    continue

      return
      end
```

APPENDIX III

TYPICAL INPUT DECK USED FOR ANALYSIS

```
*HEADING,UNSYMM
Roll pass - 150mm/min - 0.03m to 0.025m to 0.020m to 0.015m.
**
*PREPRINT,MODEL=NO,ECHO=NO,HISTORY=NO
**
**DATA CHECK
**
**
**Define nodes, node sets, elements and element sets for slab
**
*NODE
   1, 0.735, 0.00
 101, 0.985, 0.00
8001, 0.735, 0.03
8101, 0.985, 0.03
**
*NGEN,NSET=BOTTOM
1,101,1
*NGEN,NSET=TOP
8001,8101,1
*NFILL,NSET=SLAB,BIAS=1.1
BOTTOM,TOP,8,1000
*NSET,NSET=LEFT,GENERATE
1,8001,1000
*NSET,NSET=RIGHT,GENERATE
101,8101,1000
**
*ELEMENT, TYPE=CPE4R
1, 1,2,1002,1001
*ELGEN, ELSET=SLAB
1,100,1,1,8,1000,100
*ELSET, ELSET=MID
50,150,250,350,450,550,650,750
**
**Define nodes, node sets, elements, etc. for the roller
**
*NODE, NSET=REF
10000, 1.050, 0.4
**
**Rigid surface contact elements on surface of slab
**
*ELEMENT, TYPE=IRS21
50000, 8101, 8100, 10000
40000, 101, 1101, 10000
30000, 8001,7001, 10000
*ELGEN, ELSET=ROLLER
50000,  100, -1, 1
40000,  8, 1000, 1
30000, 8, -1000, 1
**
*RIGID SURFACE, ELSET=ROLLER, TYPE=SEGMENTS
START, 1.050, 0.025
CIRCL, 0.675, 0.400, 1.050, 0.4
CIRCL, 1.050, 0.775, 1.050, 0.4
CIRCL, 1.425, 0.400, 1.050, 0.4
CIRCL, 1.050, 0.025, 1.050, 0.4
```

```
**Define Material Properties
**
*SOLID SECTION, ELSET=SLAB, MATERIAL=Al1%Mg
0.5
*HOURGLASS STIFFNESS
1.3269E8
**
*MATERIAL, NAME=Al1%Mg
*USER MATERIAL, CONSTANTS=9
69.0E9, 0.3, 30.0E6, 0.0, 30.0E6, 1., 1.146581, 4.844182
100.0
*DEPVAR
51
*INTERFACE, ELSET=ROLLER
*FRICTION
0.4
**
*BOUNDARY, TYPE=DISPLACEMENT
BOTTOM, YSYMM
REF, 1, 5, 0.0
**
**
**Step 1: Spin roller and move slab into roll gap
**
*STEP, INC=1000, NLGEOM
*STATIC
1.0E-5,0.028,1.0E-10
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY, TYPE=DISPLACEMENT
REF,6,6,0.1866
RIGHT,1,1,0.07
*RESTART,WRITE,FREQ=1000
*MONITOR,NODE=8101,DOF=1,FREQUENCY=10
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**Step 2: Reduce reaction force and redefine boundaries
**
*STEP,AMP=STEP,NLGEOM,INC=1000
*STATIC
0.01,0.02
*CONTROLS, PARAMETERS=TIME INCREMENTATION
8,10, ,30, ,10
*BOUNDARY,OP=NEW
REF, 1,5,0.0
REF, 6,6,0.1866
BOTTOM, YSYMM
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
**
**
**Step 3: First roll pass
**
*STEP, INC=3000, NLGEOM,AMP=RAMP
```

```
*STATIC
0.001,0.1107,1.0E-08
*CONTROLS, PARAMETERS=TIME INCREMENTATION
8,10, ,30, ,10
*BOUNDARY, OP=NEW, TYPE=DISPLACEMENT
BOTTOM, YSYMM
REF, 1,5,0.0
REF, 6,6,0.9239
*RESTART,WRITE,FREQ=300
*MONITOR,NODE=10000,DOF=6,FREQUENCY=10
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Step 4:  Recrystallization and move slab out from under roll
**
*STEP,AMP=RAMP,NLGEOM,INC=1000
*STATIC
10.0,130.0,1.0E-8,10.0
*BOUNDARY,OP=NEW
REF,1,5,0.0
REF,6,6,0.9239
BOTTOM,YSYMM
  1,1,1,0.3735
1001,1,1,0.3735
2001,1,1,0.3735
3001,1,1,0.3735
4001,1,1,0.3736
5001,1,1,0.3736
6001,1,1,0.3739
7001,1,1,0.3742
8001,1,1,0.376
*MONITOR,NODE=1,DOF=1
*RESTART,WRITE,FREQ=5
*ELPRINT,ELSET=MID,FREQ=1
SDV33,SDV3,SDV4,SDV5,SDV9
*ELPRINT,ELSET=MID,FREQ=1
SDV33,SDV1
*ELFILE,ELSET=MID,FREQ=1
SDV
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Step 5:  Move roller down
**
*STEP,AMP=STEP,NLGEOM,INC=1000
*STATIC
0.5,1.0
*BOUNDARY,OP=NEW
REF,1,1,.0.0
REF,2,2,-0.005
REF,3,5,0.0
REF,6,6,0.9239
BOTTOM,YSYMM
```

```
  1,1,1,0.3735
1001,1,1,0.3735
2001,1,1,0.3735
3001,1,1,0.3735
4001,1,1,0.3736
5001,1,1,0.3736
6001,1,1,0.3739
7001,1,1,0.3742
8001,1,1,0.376
*RESTART,WRITE,FREQ=100
*MONITOR,NODE=10000,DOF=2
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Rol_2 - Second rollpass
**
**
**Step 6: Spin roller and move slab into roll gap
**
*STEP, AMP=RAMP, INC=1000, NLGEOM
*STATIC
1.0E-5,0.0264,1.0E-8
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY, OP=NEW, TYPE=DISPLACEMENT
REF,1,1,0.0
REF,2,2,-0.005
REF,3,5,0.0
REF,6,6,0.7479
BOTTOM,YSYMM
  1,1,1,0.3075
1001,1,1,0.3075
2001,1,1,0.3075
3001,1,1,0.3075
4001,1,1,0.3076
5001,1,1,0.3076
6001,1,1,0.3079
7001,1,1,0.3082
8001,1,1,0.31
*MONITOR,NODE=8001,DOF=1,FREQUENCY=10
*RESTART,WRITE,FREQ=50
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**Step 7: Reduce reaction force and redefine boundaries
**
*STEP,AMP=STEP,NLGEOM,INC=1000
*STATIC
0.01,0.02
*CONTROLS, PARAMETERS=TIME INCREMENTATION
,30, ,10
*BOUNDARY,OP=NEW
REF,1,1,0.0
```

```
REF,2,2,-0.005
REF,3,5,0.0
REF,6,6,0.7479
BOTTOM,YSYMM
*RESTART,WRITE,FREQ=100
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
**
**
**Step 8: Second roll pass
**
*STEP, INC=3000, NLGEOM,AMP=RAMP
*STATIC
0.001,0.1373,1.0E-08
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY, OP=NEW, TYPE=DISPLACEMENT
BOTTOM,YSYMM
REF,1,1,0.0
REF,2,2,-0.005
REF,3,5,0.0
REF,6,6,-0.1668
*RESTART,WRITE,FREQ=1000
*MONITOR,NODE=10000,DOF=6,FREQUENCY=10
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Step 9:  Recrystallization and move slab out from under roll
**
*STEP,AMP=RAMP,NLGEOM,INC=1000
*STATIC
10.0,200.0,1.0E-8,10.0
*BOUNDARY,OP=NEW
REF,1,1,0.0
REF,2,2,-0.005
REF,3,5,0.0
REF,6,6,-0.1668
BOTTOM,YSYMM
 101,1,1,0.0067
1101,1,1,0.0067
2101,1,1,0.0068
3101,1,1,0.0068
4101,1,1,0.0067
5101,1,1,0.0066
6101,1,1,0.0059
7101,1,1,0.0052
8101,1,1,0.0028
*MONITOR,NODE=101,DOF=1
*RESTART,WRITE,FREQ=20
*ELPRINT,ELSET=MID,FREQ=1
SDV33,SDV1,SDV3,SDV4
*ELPRINT,ELSET=MID,FREQ=1
SDV33,SDV5,SDV6,SDV9,SDV10
*ELFILE,ELSET=MID,FREQ=1
```

```
SDV
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Step 10:  Move roller down
**
*STEP,AMP=STEP,NLGEOM,INC=1000
*STATIC
0.5,1.0
*BOUNDARY,OP=NEW
REF,1,1,.0.0
REF,2,2,-0.01
REF,3,5,0.0
REF,6,6,-0.1668
BOTTOM,YSYMM
 101,1,1,0.0067
1101,1,1,0.0067
2101,1,1,0.0068
3101,1,1,0.0068
4101,1,1,0.0067
5101,1,1,0.0066
6101,1,1,0.0059
7101,1,1,0.0052
8101,1,1,0.0028
*MONITOR,NODE=10000,DOF=2
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Rol_3 - Third rollpass
**
**
**RESTART,READ,STEP=10,INC=2,ENDSTEP
**
**DATA CHECK
**
**
**Step 11: Spin roller and move slab into roll gap
**
*STEP, AMP=RAMP, INC=1000, NLGEOM
*STATIC
1.0E-5,0.0219.0,1.0E-8
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY, OP=NEW, TYPE=DISPLACEMENT
REF,1,1,0.0
REF,2,2,-0.01
REF,3,5,0.0
REF,6,6,-0.0217
BOTTOM,YSYMM
 101,1,1,0.0611
1101,1,1,0.0611
2101,1,1,0.0612
3101,1,1,0.0612
```

```
4101,1,1,0.0611
5101,1,1,0.0610
6101,1,1,0.0603
7101,1,1,0.0596
*MONITOR,NODE=8001,DOF=1,FREQUENCY=10
*RESTART,WRITE,FREQ=1000
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**Step 12: Reduce reaction force and redefine boundaries
**
*STEP,AMP=STEP,NLGEOM,INC=1000
*STATIC
0.0025,0.005
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY,OP=NEW
REF,1,1,0.0
REF,2,2,-0.01
REF,3,5,0.0
REF,6,6,-0.0217
BOTTOM,YSYMM
*RESTART,WRITE,FREQ=100
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*END STEP
**
**
**Step 13: Third roll pass
**
*STEP, INC=8000, NLGEOM,AMP=RAMP
*STATIC
0.001,0.1898,1.0E-08
*CONTROLS, PARAMETERS=TIME INCREMENTATION
 ,30, ,10
*BOUNDARY, OP=NEW, TYPE=DISPLACEMENT
BOTTOM,YSYMM
REF,1,1,0.0
REF,2,2,-0.01
REF,3,5,0.0
REF,6,6,1.173
*RESTART,WRITE,FREQ=1000
*MONITOR,NODE=10000,DOF=6,FREQUENCY=10
*ELPRINT,FREQ=0
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**Step 14: Recrystallization and move slab from under roll
**
*STEP,AMP=RAMP,NLGEOM,INC=1000
*STATIC
10.0,500.0,1.0E-08,100.0
*BOUNDARY,OP=NEW
REF,1,1,0.0
```

```
REF,2,2,-0.01
REF,3,5,0.0
REF,6,6,1.173
BOTTOM,YSYMM
  1,1,1,0.3221
1001,1,1,0.3221
2001,1,1,0.3220
3001,1,1,0.3219
4001,1,1,0.3222
5001,1,1,0.3234
6001,1,1,0.3254
*MONITOR,NODE=101,DOF=1
*RESTART, WRITE, FREQ=10
*ELPRINT,ELSET=MID,FREQ=2
SDV33,SDV1,SDV3,SDV4
*ELPRINT,ELSET=MID,FREQ=2
SDV33,SDV5,SDV6,SDV7,SDV8
*ELPRINT,ELSET=MID,FREQ=2
SDV33,SDV9,SDV10,SDV11,SDV12
*ELFILE,ELSET=MID,FREQ=2
SDV
*NODE PRINT,FREQ=0
*ENDSTEP
**
**
**
**
```

APPENDIX IV

DESCRIPTION OF SOLUTION DEPENDENT VARIABLE (SDV) AND PROPERTY ARRAYS USED IN ANALYSIS.

I.1 Solution Dependent Variable Array (STATEV)

No.	Symbol / Name	Description
1	ε^{pl}	Plastic Strain
2	$\dot{\varepsilon}^{pl}$	Plastic Strain Rate
3	Z	Zener-Holloman Parameter
4	d_{mean}	Average Grain Size
5-8	$X_0(i)$	Array of original volume fractions at start of each roll pass
9-12	$X_v(i)$	Array of recrystallized volume fractions during recrystallization steps
13-16	$d_0(i)$	Array of original grain sizes at start of each roll pass
17-20	$d_{rex}(i)$	Array of recrystallized grain sizes during recrystallization steps
21-24	$\varepsilon^{pl}(i)$	Array of accumulated strain for each of the corresponding volume fractions
25-28	$t_{0.5}(i)$	Array of time to 50% recrystallization for each of the corresponding volume fractions
29-32	$ratio(i)$	Array of stored original volume fractions to facilitate bulk volume fraction calculations
33	t_{tot}	Total time during each recrystallization step
34	*check*	Variable to flag whether recrystallization arrays have been initialised
35	*stime*	Time during each deformation step (roll pass)
36	*sstrain*	Strain during each deformation step (roll pass)
37	*step time*	Counts the step time of each step
38	*temperature*	Temperature
39	*step number*	Step number
40	-	Variable used to enable/disable subroutine CRYSTAL
41	-	Variable used to enable/disable subroutine INITCRYS
42	*rexvol*	Total recrystallized volume fraction over entire bulk
43	*unrexv*	Total recrystallized volume fraction over entire bulk
44-47	*ds(i)*	Current sacrificial grain size
47-51	*dr(i)*	Current recrystallizing grain size

I.2 Properties array (PROPS)

No.	Symbol	Description
1	E	Young's Modulus
2	ν	Poisson's Ratio
3	σ_y	Static Yield Strength
4	ε_y^{pl}	Plastic Strain at Yield Stress (always zero)
5	σ_1	Stress at any point 1 in the plastic region
6	ε_1^{pl}	Plastic Strain at point 1 described above
7	D	Rate Dependency constant
8	P	Rate Dependency constant
9	d_0	Initial grain size